Vector Algebra

Question1

Let $\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}, 2\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ be the position vectors of four points A, B, C and D respectively. If a point P divides AB in the ratio $2:1$ internally and a point Q divides CD in the ratio $1:2$ externally, then the ratio in which the point with position vectors $5\hat{\mathbf{i}}-6\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ divides PQ is

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Options:

A.

2 : 1

B.

$-2:1$

C.

2 : 3

D.

$-2:3$

Answer: B

Solution:

The position vectors are given as

$$a=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}, b=\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$$
$$c=2\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}, d=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

$\because P$ divides AB internally at the ratio $2:1$

$$\therefore P=\frac{2b+a}{2+1}=\frac{2(\hat{i}+\hat{j}-2\hat{k})+(\hat{i}-2\hat{j}+\hat{k})}{3}$$
$$=\hat{i}-\hat{k}$$

and Q divides CD externally in the $1:2$

$$\therefore q=\frac{\mathbf{d}-2\mathbf{c}}{1-2}=\frac{(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})-2(2\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}})}{-1}$$
$$=3(\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}})$$

Let $\mathbf{r}=5\hat{\mathbf{i}}-6\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ divides PQ in ratio $m:n$

then $\mathbf{r}=\frac{n(\hat{\mathbf{i}}-\hat{\mathbf{k}})+m(3\hat{\mathbf{i}}-3\hat{\mathbf{j}}-3\hat{\mathbf{k}})}{m+n}$

$= 5\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

then, $\frac{n+3m}{m+n} = 5, \frac{-3m}{m+n} = -6$

$\frac{-n-3m}{m+n} = -5$

By solving these equations

We get, $m : n = -2 : 1$

Hence, the ratio in which R divides PQ is $-2 : 1$

Question2

If $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}, \mathbf{b} = 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ are two vectors such that $\mathbf{r} \times \mathbf{a} = \mathbf{b} \times \mathbf{a} \cdot \mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$, then the unit vector in the direction of $\mathbf{r}$ is

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Options:

A.

$\frac{1}{\sqrt{11}}(\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$

B.

$\frac{1}{\sqrt{11}}(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$

C.

$\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

D.

$\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$

Answer: A

Solution:

$\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{b} = 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\mathbf{a} = \mathbf{b} \times \mathbf{a}$ and $\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$

So, $\mathbf{r} = \mathbf{a} + \mathbf{b}$ satisfies both equations Thus $r = \mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$= \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

then, $|r| = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11}$

Therefore, the unit vector in the direction of

$r = \frac{T}{|T|} = \frac{1}{\sqrt{11}}(\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$

Question3

If $\mathbf{a} \cdot \mathbf{b} \cdot \mathbf{c}$ are three units vectors such that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\sqrt{3}}{2}\mathbf{b} + \frac{\mathbf{c}}{\mathbf{2}}$ and α, β are the angles between $\mathbf{a}, \mathbf{c}$ and $\mathbf{a}, \mathbf{b}$ respectively, then $\alpha + \beta =$

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Options:

A.

$\frac{\pi}{2}$

B.

$\frac{7\pi}{6}$

C.

$\frac{\pi}{6}$

D.

$\frac{5\pi}{6}$

Answer: D

Solution:

$\because \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\sqrt{3}}{2}\mathbf{b} + \frac{\mathbf{c}}{2}$

and $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

so, $(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = \frac{\sqrt{3}}{2}\mathbf{b} + \frac{\mathbf{c}}{2}$

therefore, $\mathbf{a} \cdot \mathbf{c} = \frac{\sqrt{3}}{2}$

Which gives $\alpha = \pi/6$ because $\mathbf{a}$ and $\mathbf{c}$ are unit vector

and $\mathbf{a} \cdot \mathbf{b} = -\frac{1}{2}$

which gives $\beta = \frac{2\pi}{3}$

Hence, $\alpha + \beta = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6}$

Question4

Let $(x, y) \in R \times R$ and $\mathbf{a} = x\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}, \mathbf{b} = 6\hat{\mathbf{i}} - y\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ be two vectors. If

$|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = f(x)g(y), \text{ then } f(x) + g(y) - 46 = 0$

represents

Options:

A.

a pair of line

B.

an ellipse

C.

a hyperbola

D.

a circle

Answer: D

Solution:

Step 1: List the Vectors

The two vectors are:
$\mathbf{a} = x\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$
$\mathbf{b} = 6\hat{\mathbf{i}} - y\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Step 2: Use the Given Relationship

We are given that:
$|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = f(x)g(y)$

Step 3: Recognize the Identity

There is a formula: $|\mathbf{a} \times \mathbf{b}|^2 + |\mathbf{a} \cdot \mathbf{b}|^2 = |\mathbf{a}|^2 \cdot |\mathbf{b}|^2$

Step 4: Find the Lengths of the Vectors

For $\mathbf{a}$:
$|\mathbf{a}| = \sqrt{x^2 + 2^2 + (-1)^2} = \sqrt{x^2 + 4 + 1} = \sqrt{x^2 + 5}$
For $\mathbf{b}$:
$|\mathbf{b}| = \sqrt{6^2 + y^2 + 2^2} = \sqrt{36 + y^2 + 4} = \sqrt{y^2 + 40}$

Step 5: Substitute in the Formula

$|\mathbf{a}|^2 \cdot |\mathbf{b}|^2 = (x^2 + 5)(y^2 + 40)$

So, $f(x)g(y) = (x^2 + 5)(y^2 + 40)$

Step 6: Identify $f(x)$ and $g(y)$

So, $f(x) = x^2 + 5$

$g(y) = y^2 + 40$

Step 7: Use the Given Equation

$f(x) + g(y) - 46 = 0$

Step 8: Substitute for $f(x)$ and $g(y)$

$x^2 + 5 + y^2 + 40 - 46 = 0$
$x^2 + y^2 - 1 = 0$

Step 9: Final Form

So the equation is:
$x^2 + y^2 = 1$

Question5

$\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{c}$ are the position vectors of three non-collinear points on a plane. If $\alpha = [\mathbf{abc}]$ and $\mathbf{r} = \mathbf{a} \times \mathbf{b} - \mathbf{c} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}$, then $\left|\frac{\alpha}{\mathbf{r}}\right|$ represents

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Options:

A.

Ratio of areas of the triangles formed by $\mathbf{0}, \mathbf{a}, \mathbf{b}$ to $\mathbf{0}, \mathbf{bc}$

B.

Ratio of the numerical values of volume of the parallelopiped formed with $\mathbf{0}, \mathbf{a}, \mathbf{b}, \mathbf{c}$ and its height

C.

Ratio of lengths of the diagonals of the parallelopiped formed with $\mathbf{0, a, b, c}$

D.

Length of the perpendicular from origin to the plane

Answer: D

Solution:

We have, $\mathbf{r} = \mathbf{a} \times \mathbf{b} - \mathbf{c} \times \mathbf{b} - \mathbf{a} \times \mathbf{c}$

$\Rightarrow \mathbf{r} = \mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}$
$\Rightarrow \mathbf{r} \cdot \mathbf{a} = \alpha \cdot \mathbf{l}, \mathbf{r} \cdot \mathbf{b} = \alpha, \mathbf{r} \cdot \mathbf{c} = \alpha$

Equation of plane passing through $A(\mathbf{a}), B(\mathbf{b})$ and $C(\mathbf{c})$ is

$n = AB \times BC$
$= (b - a) \times (c - b) = b \times c - a \times c + a \times b$
$= a \times b + b \times c + c \times a = r$

Equation of plane

$$(\mathbf{r}_1 - \mathbf{a}) \cdot \mathbf{n} = 0$$
$$\Rightarrow \mathbf{r}_1 \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n} \Rightarrow \mathbf{r}_1 \cdot \mathbf{r} = \mathbf{a} \cdot \mathbf{r}$$
$$\mathbf{r}_1 \cdot \mathbf{r} = \alpha$$
$$\mathbf{r}_1 \cdot \frac{\mathbf{r}}{|\mathbf{r}|} = \frac{|\alpha|}{|\mathbf{r}|} \Rightarrow \mathbf{r} \cdot \hat{\mathbf{n}} = p$$

$\therefore \frac{|\alpha|}{|\mathbf{r}|}$ is perpendicular distance from origin to plane.

Question6

If $P=(\mathbf{a}\times\hat{\mathbf{i}})^2+(\mathbf{a}\times\hat{\mathbf{j}})^2+(\mathbf{a}\times\hat{\mathbf{k}})^2$ and $Q=(\mathbf{a}\cdot\hat{\mathbf{i}})^2+(\mathbf{a}\cdot\hat{\mathbf{j}})^2+(\mathbf{a}\cdot\hat{\mathbf{k}})^2$, then

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Options:

A.

$P=Q$

B.

$P=2Q$

C.

$P=3Q$

D.

$P=4Q$

Answer: B

Solution:

Let $\mathbf{a}=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$

$$\begin{aligned}
\therefore\quad & \mathbf{a}\times\hat{\mathbf{i}}=-b\hat{\mathbf{k}}+c\hat{\mathbf{j}}\Rightarrow\mathbf{a}\times\hat{\mathbf{j}}=a\hat{\mathbf{k}}-c\hat{\mathbf{i}}\\
\Rightarrow\quad & \mathbf{a}\times\hat{\mathbf{k}}=-a\hat{\mathbf{j}}+b\hat{\mathbf{i}}\\
\therefore\quad & (\mathbf{a}\times\hat{\mathbf{i}})^2=b^2+c^2\\
& (\mathbf{a}\times\hat{\mathbf{j}})^2=a^2+c^2\\
& (\mathbf{a}\times\hat{\mathbf{k}})^2=a^2+b^2\\
\because\quad & P=2\left(a^2+b^2+c^2\right)
\end{aligned}$$

Now, $Q=a^2+b^2+c^2$

$\therefore\quad P=2Q$

Question7

$\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}},\mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}},\mathbf{c}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ are three vectors. If $\mathbf{r}$ is a vector such that $\mathbf{r}\cdot\mathbf{a}=0,\mathbf{r}\cdot\mathbf{c}=3$ and $\begin{bmatrix}\mathbf{r} & \mathbf{a}\\ \mathbf{b} & \end{bmatrix}=0$, then $|\mathbf{r}|=$

Options:

A.

$\sqrt{2}$

B.

$\sqrt{3}$

C.

3

D.

7

Answer: A

Solution:

A Given,

$\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}, \mathbf{b}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}},$

$\mathbf{c}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\mathbf{r}\cdot\mathbf{a}=0 \Rightarrow [\mathbf{r}\cdot\mathbf{ab}]=0 \Rightarrow \mathbf{r}\cdot(\mathbf{a}\times\mathbf{b})=0$

$\therefore \quad \mathbf{r}=\lambda(\mathbf{a}\times(\mathbf{a}\times\mathbf{b})$

$=\lambda[(\mathbf{a}\cdot\mathbf{b})\mathbf{a}-\mathbf{a}\cdot\mathbf{a}(\mathbf{b})]$

$=\lambda(9\mathbf{a}-6\mathbf{b})=3\lambda(3\mathbf{a}-2\mathbf{b})$

$\mathbf{r}=k(3\mathbf{a}-2\mathbf{b})$

$\mathbf{r}\cdot\mathbf{c}=k(3\mathbf{a}\cdot\mathbf{c}-2\mathbf{b}\cdot\mathbf{c})$

$\Rightarrow \quad 3=k[3(2-(-2)-2(2-2-3)]$

$\Rightarrow \quad 3=k(-3+6) \Rightarrow k=7$

$\therefore \quad \mathbf{r}=\hat{\mathbf{i}}-\hat{\mathbf{j}} \Rightarrow |\mathbf{r}|=\sqrt{2}$

Question8

In a right angled triangle, if the position vector of the vertex having the right angle is $-3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and the position vector of the mid-point of its hypotenuse is $6\hat{\mathbf{i}}+2\hat{\mathbf{j}}+5\hat{\mathbf{k}}$, then the position vector of its centroid is

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Options:

A.

$3\hat{i}+3\hat{j}+4\hat{k}$

B.

$3\hat{i}+3\hat{j}+3\hat{k}$

C.

$\frac{3\hat{i}+7\hat{j}+7\hat{k}}{2}$

D.

$4\hat{j}+3\hat{k}$

Answer: A

Solution:

The position vector of $\mathbf{a}$ is

$\mathbf{a}=-3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

Let, M be the mid-point of the hypotenuse BC.

So, the position vector of $\mathbf{M}$ is

$\mathbf{m}=6\hat{\mathbf{i}}+2\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

Let the centroid be G and its position vector be $\mathbf{g}$.

So, $\mathbf{g}=\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$ $\quad\quad\quad \ldots(i)$

Since, the mid-point of the hypotenuse of a right triangle is equidistant from all three vertices.

So, M is the circumcenter of a triangle and $\mathbf{m}=\frac{\mathbf{b}+\mathbf{c}}{2}$

$\Rightarrow \mathbf{b}+\mathbf{c}=2\mathbf{m}$

Substituting this value in Eq. (i), we get

$$\begin{aligned}
g &= \frac{\mathbf{a}+2\mathbf{m}}{3} \\
\Rightarrow g &= \frac{(-3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}})+2(6\hat{\mathbf{i}}+2\hat{\mathbf{j}}+5\hat{\mathbf{k}})}{3} \\
&= \frac{-3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}+12\hat{\mathbf{i}}+4\hat{\mathbf{j}}+10\hat{\mathbf{k}}}{3} \\
&= \frac{9\hat{\mathbf{i}}+9\hat{\mathbf{j}}+12\hat{\mathbf{k}}}{3} \\
&= 3\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}
\end{aligned}$$

So, the position vector of centroid is

$3\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

Question9

If the position vectors of the vertices A, B, C of a triangle are $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}, \hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}, 5(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ respectively, then the magnitude of the altitude drawn from A on to the side BC is

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Options:

A.

$\frac{4\sqrt{5}}{3}$

B.

$\frac{5\sqrt{5}}{3}$

C.

$\frac{7\sqrt{5}}{3}$

D.

$\frac{8\sqrt{5}}{3}$

Answer: A

Solution:

Given,

$$\begin{aligned}&\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}, \mathbf{B} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}} \text{ and}\\ &\quad \mathbf{C} = 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 5\hat{\mathbf{k}}\\ &\text{So, } \mathbf{BC} = \mathbf{C} - \mathbf{B}\\ &\qquad = 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 5\hat{\mathbf{k}} - \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}\\ &\qquad = 4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}\\ &\mathbf{BA} = \mathbf{A} - \mathbf{B} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}} - \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}\\ &\quad = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}\end{aligned}$$

$$\text{Now, } \mathbf{BA} \times \mathbf{BC} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -2 \\ 4 & 2 & 4 \end{vmatrix}$$

$$\begin{aligned}&= (4+4)\hat{\mathbf{i}} - (8+8)\hat{\mathbf{j}} + (4-4)\hat{\mathbf{k}}\\ &= 8\hat{\mathbf{i}} - 16\hat{\mathbf{j}}\end{aligned}$$

$$\text{And } |\mathbf{BA} \times \mathbf{BC}| = \sqrt{8^2 + (-16)^2}$$

$$\begin{aligned}&= \sqrt{64 + 256}\\ &= \sqrt{320} = 8\sqrt{5}\end{aligned}$$

$$\begin{aligned}&\text{So, area of triangle } = \frac{1}{2}|\mathbf{BA} \times \mathbf{BC}|\\ &\qquad = \frac{1}{2} \times 8\sqrt{5} = 4\sqrt{5}\\ &Now,\ |\mathbf{BC}| = \sqrt{4^2 + 2^2 + 4^2}\\ &\qquad = \sqrt{16 + 4 + 16} = \sqrt{36} = 6\end{aligned}$$

Let, the magnitude of the altitude h from A to BC is given by

$$\begin{aligned}&\text{Area } = \frac{1}{2} \times \text{ base } \times \text{ height}\\ &\Rightarrow \quad 4\sqrt{5} = \frac{1}{2} \times |BC| \times h\\ &\Rightarrow \quad 4\sqrt{5} = \frac{1}{2} \times 6 \times h\\ &\Rightarrow \quad h = \frac{4\sqrt{5}}{3}\end{aligned}$$

Question10

If the vectors $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $p\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ are coplanar, then the unit vector in the direction of the vector $9p\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ is

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Options:

A.

$\frac{1}{6}(2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

B.

$\frac{1}{\sqrt{57}}(5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

C.

$\frac{1}{\sqrt{68}}(6\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

D.

$\frac{1}{9}(-7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

Answer: D

Solution:

Let $\mathbf{a} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$,

$\mathbf{b} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{c} = p\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now, $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & 4 & -3 \\ -1 & 2 & 3 \\ p & -2 & 1 \end{vmatrix} = 0$

$$\begin{aligned} &\Rightarrow \quad 2(2+6) - 4(-1-3p) - 3(2-2p) = 0 \\ &\Rightarrow \quad 16 + 4 + 12p - 6 + 6p = 0 \\ &\Rightarrow \quad 18p + 14 = 0 \\ &\Rightarrow \quad p = \frac{-14}{18} = \frac{-7}{9} \end{aligned}$$

Now, $\mathbf{v} = 9p\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$= 9\left(\frac{-7}{9}\right)\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$= -7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

Now, unit vector

$$\begin{aligned} &\hat{\mathbf{u}} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{-7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{\sqrt{(-7)^2 + (-4)^2 + 4^2}} \\ \Rightarrow \quad &\hat{\mathbf{u}} = \frac{-7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{9} \\ &\quad = \frac{-7}{9}\hat{\mathbf{i}} - \frac{4}{9}\hat{\mathbf{j}} + \frac{4}{9}\hat{\mathbf{k}} \end{aligned}$$

Question11

Let $\mathbf{a} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\mathbf{b}$ be two perpendicular vectors in the XOY-plane. A vector $\mathbf{c}$ in the same plane and having projections 1 and 2 respectively on $\mathbf{a}$ and $\mathbf{b}$ is

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Options:

A.

$\hat{i} + 2\hat{j}$

B.

$2\hat{i} + \hat{j}$

C.

$\hat{i} - 2\hat{j}$

D.

$2\hat{i} - \hat{j}$

Answer: D

Solution:

Let $\mathbf{b} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$.

$$\begin{aligned}
\therefore \quad & \mathbf{a} \cdot \mathbf{b} = 0 \\
\Rightarrow \quad & (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (x\hat{\mathbf{i}} + y\hat{\mathbf{j}}) = 0 \\
\Rightarrow \quad & 4x + 3y = 0 \\
\Rightarrow \quad & \tfrac{x}{3} = \tfrac{y}{-4} = \lambda \\
\Rightarrow \quad & x = 3\lambda, y = -4\lambda \\
\therefore \quad & \mathbf{b} = \lambda(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})
\end{aligned}$$

Let the required vector be $\mathbf{c} = c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{j}}$ then the projection of $\mathbf{c}$ on $\mathbf{a}$ and b are $\frac{\mathbf{c}\cdot\mathbf{a}}{|\mathbf{a}|}$ and $\frac{\mathbf{c}\cdot\mathbf{b}}{|\mathbf{b}|}$ respectively.

$$\begin{aligned}
\therefore \quad & \frac{\mathbf{c} \cdot \mathbf{a}}{|\mathbf{a}|} = 1 \text{ and } \frac{\mathbf{c} \cdot \mathbf{b}}{|\mathbf{b}|} = 2 \\
\Rightarrow \quad & \frac{\left(c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{j}}\right) \cdot (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})}{\sqrt{4^2 + 3^2}} = 1 \\
\Rightarrow \quad & 4c_1 + 3c_2 = 5 \qquad \ldots(i) \\
\text{and} \quad & \frac{\mathbf{c} \cdot \mathbf{b}}{|\mathbf{b}|} = 2 \\
\Rightarrow \quad & \frac{\left(c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{j}}\right) \cdot (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})}{\sqrt{3^2 + (-4)^2}} = 2 \\
\Rightarrow \quad & 3c_1 - 4c_2 = 10 \qquad \ldots(ii)
\end{aligned}$$

Solving Eq. (i) and (ii), we get

$c_1 = 2, c_2 = -1$

So, the required vector

$= c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{j}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}}$

Question12

If $\mathbf{a} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{b} = -\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ are two vectors, then the vector of magnitude 28 units in the direction of the vector $\mathbf{a} - \mathbf{b}$ is

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Options:

A.

$3\hat{i} + 6\hat{j} - 2\hat{k}$

B.

$12\hat{\mathbf{i}} - 24\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

C.

$3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

D.

$12\hat{i} + 24\hat{j} - 8\hat{k}$

Answer: B

Solution:

$\mathbf{a} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}, \mathbf{b} = -\hat{\mathbf{i}} + 3\mathbf{j} + 3\hat{\mathbf{k}}$

So, $\mathbf{a} - \mathbf{b} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}} + \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$= 3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

and $|\mathbf{a} - \mathbf{b}| = \sqrt{3^2 + (-6)^2 + 2^2} = 7$

Unit vector in (a - b)

$= \frac{1}{7}(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

thus, the vector of magnitude 28 units

$$= 28\left(\frac{1}{7}(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 4(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})\right)$$
$$= 12\hat{\mathbf{i}} - 24\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

Question13

If $\bar{a}$ is a unit vector, then

$|\mathbf{a} \times \hat{\mathbf{i}}|^2 + |\mathbf{a} \times \hat{\mathbf{j}}|^2 + |\mathbf{a} \times \hat{\mathbf{k}}|^2 =$

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Options:

A.

4

B.

1

C.

0

D.

2

Answer: D

Solution:

Let, $\mathbf{a} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$$then, \mathbf{a} \times \mathbf{i} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-z) + \hat{\mathbf{k}}(0-y)$$

$$= z\hat{\mathbf{j}} - y\hat{\mathbf{k}}$$

So, $|\mathbf{a} \times \mathbf{i}|^2 = z^2 + y^2$

Similarly, $|\mathbf{a} \times \hat{\mathbf{j}}|^2 = x^2 + z^2$

and $|\mathbf{a} \times \hat{\mathbf{k}}|^2 = x^2 + y^2$

thus, $|\mathbf{a} \times \hat{\mathbf{i}}|^2 + |\mathbf{a} \times \hat{\mathbf{j}}|^2 + |\mathbf{a} \times \hat{\mathbf{k}}|^2$

$= 2\left(x^2 + y^2 + z^2\right)$

$\because$ $\mathbf{a}$ is unit vector

So, $x^2 + y^2 + z^2 = 1$

Hence, $|\mathbf{a} \times \hat{\mathbf{i}}|^2 + |\mathbf{a} \times \hat{\mathbf{j}}|^2 + |\mathbf{a} \times \hat{\mathbf{k}}|^2$

$= 2 \times 1 = 2$

Question14

If $\mathbf{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, $\mathbf{b} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, $\mathbf{c} = 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{d} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ are four vectors, then $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) =$

Options:

A.

$18\hat{i}+6\hat{j}+30\hat{k}$

B.

$8\hat{\mathbf{i}}-3\hat{\mathbf{j}}+8\hat{\mathbf{k}}$

C.

$19\hat{i}-5\hat{j}+21\hat{k}$

D.

$27\hat{i}-8\hat{j}+29\hat{k}$

Answer: A

Solution:

$$(\mathbf{a}\times\mathbf{b})=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & -2 & -3\\ -2 & 3 & 4\end{vmatrix}$$
$$=\hat{\mathbf{i}}(-8+9)-\hat{\mathbf{j}}(4-6)+\hat{\mathbf{k}}(3-4)$$
$$=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$$
$$\text{and }(\mathbf{c}\times\mathbf{d})=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 5 & -4 & 3\\ 3 & 1 & 5\end{vmatrix}$$

$$=\hat{\mathbf{i}}(-20-3)-\hat{\mathbf{j}}(25-9)+\hat{\mathbf{k}}(5+12)$$
$$=-23\hat{\mathbf{i}}-16\hat{\mathbf{j}}+17\hat{\mathbf{k}}$$
$$\text{Thus, }(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d})=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 2 & -\\ -23 & -16 & 7\end{vmatrix}$$
$$=\hat{\mathbf{i}}(34-16)-\hat{\mathbf{j}}(17-23)+\hat{\mathbf{k}}(-16+4)$$
$$=18\hat{\mathbf{i}}+6\hat{\mathbf{j}}+30\hat{\mathbf{k}}$$

Question15

$3\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}+5\hat{\mathbf{j}}$ are the position vectors of three non-collinear points A, B, C respectively. If the perpendicular drawn from C onto $\mathbf{AB}$ meets $\mathbf{AB}$ at the point $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$, then $a+b+c=$

Options:

A.

5

B.

3

C.

7

D.

9

Answer: C

Solution:

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = (2-3)\hat{\mathbf{i}} + (0-1)\hat{\mathbf{j}} + (1-1)\hat{\mathbf{k}} = -\hat{\mathrm{i}} - \hat{\mathbf{j}}$

So, the direction vector of lin ϵAB is

$$\begin{aligned} &\mathbf{d} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} \\ \text{let } \mathbf{P} &= \mathbf{A} + \lambda\mathbf{d} \\ &= (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + \lambda(-\hat{\mathbf{i}} - \hat{\mathbf{j}}) \\ &= (3-\lambda)\hat{\mathbf{i}} + (1-\lambda)\hat{\mathbf{j}} + \hat{\mathbf{k}} \\ \text{and } \mathbf{PC} &= \mathbf{C} - \mathbf{P} = (\hat{\mathbf{i}} + 5\hat{\mathbf{j}}) \\ &-[(3-\lambda)\hat{\mathbf{i}} + (1-\lambda)\hat{\mathbf{j}} + \hat{\mathbf{k}}) \\ &= (\lambda - 2\hat{\mathbf{i}} + (4+\lambda)\hat{\mathbf{j}} - \hat{\mathbf{k}} \end{aligned}$$

As, given in the question

$$\begin{aligned} &\because \mathrm{PC} \perp \mathrm{AB} \\ &\Rightarrow \mathrm{PC} \cdot \mathrm{AB} = 0 \\ &\Rightarrow (\lambda - 2)(-1) + (4+\lambda)(-1) + (-1)(0) = 0 \\ &\Rightarrow -\lambda + 2 - 4 - \lambda = 0 \\ &\Rightarrow \lambda = -1 \\ \therefore \quad & \mathrm{P} = (3-\lambda)\hat{\mathrm{i}} + (1-\lambda)\hat{\mathrm{j}} + \hat{\mathrm{k}} \\ &\quad = (3+1)\hat{\mathrm{i}} + (1+1)\hat{\mathrm{j}} + \hat{\mathrm{k}} \\ &\quad = 4\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}} \end{aligned}$$

therefore, $a = 4, b = 2$ and $c = 1$

Hence, $a + b + c = 4 + 2 + 1 = 7$

Question16

If the vectors $2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + l\hat{\mathrm{k}}, -3\hat{\mathrm{i}} - 2\hat{\mathrm{j}} - 4l\hat{\mathrm{k}}$ and $\hat{\mathrm{i}} - \hat{\mathrm{j}} + 3/\hat{\mathrm{k}}$ form a right-angled triangle for a positive value of l, then the length of its hypotenuse is

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Options:

A.

$\sqrt{\frac{40}{3}}$

B.

$\sqrt{\frac{55}{3}}$

C.

$\sqrt{\frac{65}{3}}$

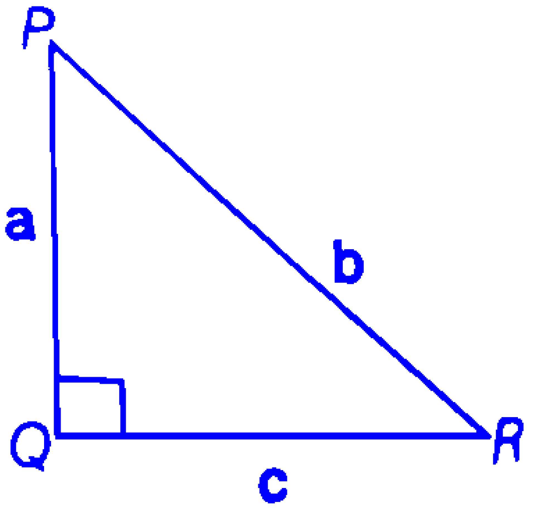
D.

$\sqrt{\frac{59}{3}}$

Answer: B

Solution:

$$\begin{aligned} \text{Let } \mathbf{a} &= 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + l\hat{\mathbf{k}} \\ \mathbf{b} &= -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4l\hat{\mathbf{k}} \\ \mathbf{c} &= \hat{\mathbf{i}} - \hat{\mathbf{j}} + 3l\hat{\mathbf{k}} \end{aligned}$$



For right angled Δ;

$$|\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{c}|^2$$

$$\Rightarrow \quad (9 + 4 + 16l^2) = (4 + 9 + l^2) + (1 + 1 + 9l^2)$$

$$\Rightarrow \quad l^2 = \frac{1}{3} \Rightarrow l = \pm\frac{1}{\sqrt{3}} \text{ but } l > 0$$

$$\therefore \quad l = \frac{1}{\sqrt{3}}$$

So, its hypotenuse $\mathbf{b} = -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \dfrac{4}{\sqrt{3}}\hat{\mathbf{k}}$

$$\therefore \quad |\mathbf{b}| = \sqrt{9 + 4 + \frac{16}{3}}$$

$$= \sqrt{\frac{27 + 12 + 16}{3}} = \sqrt{\frac{55}{3}}$$

Question17

A unit vector that is perpendicular to the vector $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and coplanar with the vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ is

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Options:

A.

$\frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}$

B.

$\frac{3\hat{i}+2\hat{j}-2\hat{k}}{\sqrt{17}}$

C.

$\frac{2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}}{3}$

D.

$\frac{3\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{17}}$

Answer: C

Solution:

Let $\mathbf{a}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$,

$\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}, \mathbf{c}=2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$

So, a vector which is perpendicular to a and coplanar with $\mathbf{b}\times\mathbf{c}=\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$

$$=(\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$
$$=0\cdot\mathbf{b}-(-1)\mathbf{c}=\mathbf{c}=2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

$$\text{So, its unit vector } =\frac{2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{2^2+2^2+(-1)^2}}$$
$$=\frac{2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}}{3}$$

Question18

If the vectors $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}}, \hat{\mathbf{i}}+4\hat{\mathbf{j}}+\hat{\mathbf{k}}, 4\hat{\mathbf{i}}+p\hat{\mathbf{j}}+\hat{\mathbf{k}}$ are coplanar, then $p=$

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Options:

A.

53

B.

37

C.

43

D.

59

Answer: C

Solution:

Since, given vectors are coplanar.

So, its determinant value $= 0$.

$$\Rightarrow \begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & 1 \\ 4 & p & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2(4-p)+1(1-4)+3(p-16)=0$$

$$\Rightarrow \quad p=43$$

Question19

If the magnitudes of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a}+\mathbf{b}$ are respectively 3,4 and 5 , then the magnitude of $\mathbf{a}-\mathbf{b}$ is

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Options:

A.

3

B.

4

C.

6

D.

5

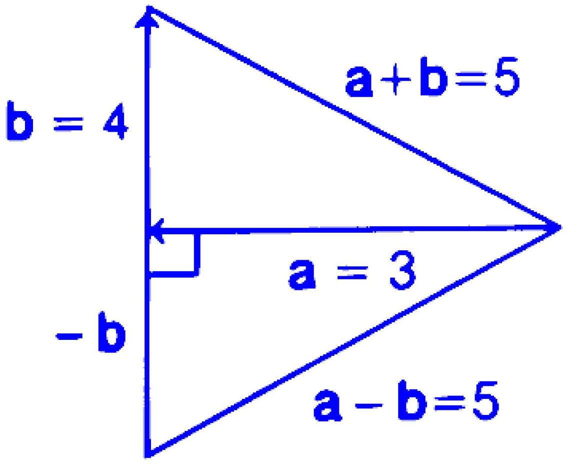
Answer: D

Solution:

(Logically) given that $|\mathbf{a}| = 3$,

$|\mathbf{b}| = 4, |\mathbf{a} + \mathbf{b}| = 5$

So, $|\mathbf{a} - \mathbf{b}| = 5$



Question20

If $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are the position vectors of four points A, B, C, D respectively, then the shortest distance between the lines AB and CD is

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Options:

A.

$\frac{1}{6}$

B.

$\frac{7}{3}$

C.

$\frac{1}{3}$

D.

$\frac{7}{6}$

Answer: C

Solution:

Direction vector of line AB is

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Similarly, direction vector of line CD is

$\text{CD} = \text{OD} - \text{OC} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
$= 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

and $\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = -\hat{\mathbf{i}} + 0 \cdot \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore$ Shortest distance (S.D)

$$= \left| \frac{\mathbf{AC} \cdot (\mathbf{AB} \times \mathbf{CD})}{|\mathbf{AB} \times \mathbf{CD}|} \right|$$

$$\mathbf{AB} \times \mathbf{CD} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -2 & 1 & 2 \\ 2 & -2 & 0 \end{vmatrix}$$

$$= 4\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

and $\mathbf{AC} \cdot (\mathbf{AB} \times \mathbf{CD}) = 2$

Also, $|\mathbf{AB} \times \mathbf{CD}| = \sqrt{16 + 16 + 4} = 4$

Hence, S.D. $= \left|\frac{2}{6}\right| = \frac{1}{3}$

Question21

A line segment PQ has the length 63 and direction ratios $(3, -2, 6)$. If this line makes an obtuse angle with X-axis, then the components of the vector $\mathbf{PQ}$ are

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Options:

A.

$7, 8, -4$

B.

$-7, 8, -4$

C.

$27, -18, 54$

D.

$-27, 18, -54$

Answer: D

Solution:

Logically, the vector must be proportional to $(3, -2, 6)$ with magnitude 63.

So, $PQ = 9(3, -2, 6) = (27, -18, 54) \rightarrow$ Component form.

The lines makes an obtuse angle with the X-axis.

$\therefore$ Components of the vector PQ are $-27, 18, -54$

Question22

The points in the argand plane represented by the complex numbers $4\hat{\mathrm{i}}+\hat{\mathrm{j}}+3\hat{\mathrm{k}}, 6\hat{\mathrm{i}}-2\hat{\mathrm{j}}-3\hat{\mathrm{k}}$ and $\hat{\mathrm{i}}-\hat{\mathrm{j}}-3\hat{\mathrm{k}}$ form

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Options:

A.

a right-angled triangle

B.

a right-angled isosceles triangle

C.

an equilateral triangle

D.

an isosceles triangle

Answer: D

Solution:

$4\hat{\mathrm{i}}+\hat{\mathrm{j}}+3\hat{\mathrm{k}}, 6\hat{\mathrm{i}}-2\hat{\mathrm{j}}-3\hat{\mathrm{k}}, \hat{\mathrm{i}}-\hat{\mathrm{j}}-3\hat{\mathrm{k}}$

$\Rightarrow$ points are $P(4,1,3), Q(6,-2,-3)$ and $R(1,-1,-3)$

$$\begin{aligned}\Rightarrow PQ &= \sqrt{(6-4)^2+(-3)^2+(-6)^2}\\ &= \sqrt{4+9+36}=\sqrt{49}=7\\ \Rightarrow QR &= \sqrt{(1-6)^2+(-1+2)^2+(-3+3)^2}\\ &= \sqrt{(-5)^2+(1)^2+0}=\sqrt{26}\\ \Rightarrow PR &= \sqrt{(1-4)^2+(-1-1)^2+(-3-3)^2}\\ &= \sqrt{9+4+36}=\sqrt{49}=7\end{aligned}$$

$\Rightarrow PQ = PR = 7$ units

$\Rightarrow \triangle PQR$ is an isosceles triangle.

Question23

If the vector $\hat{\mathrm{i}}-7\hat{\mathrm{j}}+2\hat{\mathrm{k}}$ is along the internal bisector of the angle between the vectors a and $-2\hat{\mathrm{i}}-\hat{\mathrm{j}}+2\hat{\mathrm{k}}$ and the unit vector along a is $x\hat{\mathrm{i}}+y\hat{\mathrm{j}}+z\hat{\mathrm{k}}$ then, $x=$

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Options:

A.

0

B.

$\frac{7}{9}$

C.

$-\frac{1}{9}$

D.

$\frac{5}{3}$

Answer: B

Solution:

Magnitude of the vector $-2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$\Rightarrow |\mathbf{b}|=\sqrt{4+1+4}=\sqrt{9}=3$

$\Rightarrow$ Unit vector $\hat{\mathbf{b}}=\frac{-2\hat{\mathbf{i}}}{3}-\frac{1}{3}\hat{\mathbf{j}}+\frac{2}{3}\hat{\mathbf{k}}$

Now, let $\mathbf{c}=\hat{\mathbf{i}}-7\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$|\mathbf{c}|=\sqrt{54}=3\sqrt{6}$

$\hat{\mathbf{c}}=\frac{1}{3\sqrt{6}}\hat{\mathbf{i}}-\frac{7}{3\sqrt{6}}\hat{\mathbf{j}}+\frac{2}{3\sqrt{6}}\hat{\mathbf{k}}$

$\Rightarrow \mathbf{c}=\mathbf{a}+\mathbf{b}\Rightarrow \mathbf{a}=\mathbf{c}-\mathbf{b}$

$=\frac{(1+2\sqrt{6})\hat{\mathbf{i}}}{3\sqrt{6}}+\frac{(-7+\sqrt{6})\hat{\mathbf{j}}}{3\sqrt{6}}+\frac{(2-2\sqrt{6})\hat{\mathbf{k}}}{3\sqrt{6}}$

Compaire with $\mathbf{a}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$

$\Rightarrow x=\frac{1+2\sqrt{6}}{3\sqrt{6}}=\frac{\sqrt{6}+12}{18}\cong\frac{7}{9}$

$\Rightarrow x=\frac{7}{9}$

Question24

If $\mathbf{a}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+6\hat{\mathbf{k}}$; $\mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\mathbf{c}=3\hat{\mathbf{j}}-\hat{\mathbf{k}}$, then $\mathbf{a}\times\mathbf{b}\times\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}=$

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Options:

A.

$20\hat{i}+3\hat{j}-4\hat{k}$

B.

$20\hat{i}-3\hat{j}+4\hat{k}$

C.

$3\hat{i}+20\hat{j}-4\hat{k}$

D.

$4\hat{i}+20\hat{j}-3\hat{k}$

Answer: A

Solution:

$$\begin{aligned}&\mathbf{a}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+6\hat{\mathbf{k}}, \mathbf{b}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}},\\&\mathbf{c}=3\hat{\mathbf{j}}-\hat{\mathbf{k}}\\&\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\2 & -1 & 6\\1 & -1 & 1\end{vmatrix}\\&\ =\hat{\mathbf{i}}(-1+6)-\hat{\mathbf{j}}(2-6)+\hat{\mathbf{k}}(-2+1)\\&\ =5\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}\\&\mathbf{b}\times\mathbf{c}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\1 & -1 & 1\\0 & 3 & -1\end{vmatrix}\\&\ =\hat{\mathbf{i}}(1-3)-\hat{\mathbf{j}}(-1-0)+\hat{\mathbf{k}}(3-0)\\&\ =-2\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}\end{aligned}$$

$$\begin{aligned}\mathbf{c}\times\mathbf{a}&=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\0 & 3 & -1\\2 & -1 & 6\end{vmatrix}\\&=\hat{\mathbf{i}}(18-1)-\hat{\mathbf{j}}(+2)+\hat{\mathbf{k}}(-6)\\&=17\hat{\mathbf{i}}-2\hat{\mathbf{j}}-6\hat{\mathbf{k}}\\\mathbf{a}\times\mathbf{b}&+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\\&=5\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}+(-2\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}})+17\hat{\mathbf{i}}-2\hat{\mathbf{j}}-6\hat{\mathbf{k}}\\&=20\hat{\mathbf{i}}+3\hat{\mathbf{j}}-4\hat{\mathbf{k}}\end{aligned}$$

Question25

Let $\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ be two vectors. If $\mathbf{c}^{\text{is}}$ vector such that $\mathbf{a}\cdot\mathbf{c}=|\mathbf{c}|, |\mathbf{c}-\mathbf{a}|=2\sqrt{2}$ and the angle between $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}$ is 30°, then $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=$

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Options:

A.

$\frac{2}{3}$

B.

$\frac{3}{2}$

C.

2

D.

3

Answer: B

Solution:

$\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$

$\mathbf{a}\cdot\mathbf{c}=|\mathbf{c}|, |\mathbf{c}-\mathbf{a}|=2\sqrt{2}$

$\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}$ is 30°.

$$|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=|\mathbf{a}\times\mathbf{b}||\mathbf{c}|\sin\theta$$
$$=|\mathbf{a}\times\mathbf{b}||\mathbf{c}|\times\frac{1}{2}$$
$$\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}, \mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$$
$$\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & 1 & -2\\ 1 & 1 & 0\end{vmatrix}=\hat{\mathbf{i}}(2)-\hat{\mathbf{j}}(2)+\hat{\mathbf{k}}(1)$$
$$=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$$
$$|\mathbf{a}\times\mathbf{b}|=\sqrt{4+4+1}=3$$
$$\mathbf{a}\cdot\mathbf{c}=|\mathbf{c}|$$
$$\Rightarrow|\mathbf{a}||\mathbf{c}|\cos\alpha=|\mathbf{c}|$$

$$\cos\alpha=\frac{1}{3}$$
$$|\mathbf{c}-\mathbf{a}|=2\sqrt{2}$$
$$\Rightarrow|\mathbf{c}-\mathbf{a}|^{2}=8$$
$$\Rightarrow|\mathbf{c}|^{2}+|\mathbf{a}|^{2}-2|\mathbf{c}||\mathbf{a}|\cos\alpha=8$$
$$\Rightarrow|\mathbf{c}|^{2}+9-2|\mathbf{c}|3\times\frac{1}{3}=8$$
$$\Rightarrow|\mathbf{c}|^{2}-2|\mathbf{c}|+1=0$$
$$\Rightarrow(|\mathbf{c}|-1)^{2}=0\Rightarrow|\mathbf{c}|=1$$
$$|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=|\mathbf{a}\times\mathbf{b}||\mathbf{c}|\sin\theta$$
$$=3\times1\times\sin 30^{\circ}$$
$$=3\times1\times\frac{1}{2}=\frac{3}{2}$$

Question26

$$(\mathbf{a}+2\mathbf{b}-\mathbf{c})\cdot(\mathbf{a}-\mathbf{b})\times(\mathbf{a}-\mathbf{b}-\mathbf{c})=$$

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Options:

A.

[abc]

B.

$3[\mathrm{abc}]$

C.

$[\mathrm{abc}]^2$

D.

$2[abc]$

Answer: B

Solution:

$$(a+2b-c)\{(a-b)\times(a-b-c)\}$$
$$=(a+2b-c)[a\times a-a\times b-a\times c-b\times a+b\times b+b\times c]$$

$$=(\mathbf{a}+2\mathbf{b}-\mathbf{c})[-\mathbf{a}\times\mathbf{b}-\mathbf{a}\times\mathbf{c}-\mathbf{b}\times\mathbf{a}+\mathbf{b}\times\mathbf{c}]$$
$$=(\mathbf{a}+2\mathbf{b}-\mathbf{c})[\mathbf{b}\times\mathbf{a}+\mathbf{c}\times\mathbf{a}+\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}]$$

$$\begin{aligned}=&\mathbf{a}\cdot(\mathbf{b}\times\mathbf{a})+\mathbf{a}\cdot(\mathbf{c}\times\mathbf{a})+\mathbf{a}\cdot(\mathbf{a}\times\mathbf{b})\\ &+\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})\\ &+2\mathbf{b}\cdot(\mathbf{b}\times\mathbf{a})+2\mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})+2\mathbf{b}(\mathbf{a}\times\mathbf{b})\\ &+2\mathbf{b}\cdot(\mathbf{b}\times\mathbf{c})\\ &-\mathbf{c}\cdot(\mathbf{b}\times\mathbf{a})-\mathbf{c}(\mathbf{c}\times\mathbf{a})-\mathbf{c}(\mathbf{a}\times\mathbf{b})-\mathbf{c}(\mathbf{b}\times\mathbf{c})\\ =&[\mathbf{abc}]+2[\mathbf{bca}]-[\mathbf{cba}]-[\mathbf{cab}]\\ =&[\mathbf{abc}]+2[\mathbf{abc}]+[\mathbf{abc}]-[\mathbf{abc}]=3[\mathbf{abc}]\end{aligned}$$

Question27

Points P and Q are given by $\mathbf{OP}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\mathbf{OQ}=-\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$. A line along the vector $\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ passes through the point P and another line along the vector $\mathbf{b}=\hat{\mathbf{j}}-\hat{\mathbf{k}}$ passes through the point Q. If a line along the vector $\mathbf{c}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ intersects both the lines along the vectors a and b at L and M, respectively, then $\mathbf{PM}=$

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Options:

A.

$\hat{i}-\hat{j}+2\hat{k}$

B.

$4\hat{i}+4\hat{j}$

C.

$-2\hat{\mathbf{i}}+10\hat{\mathbf{j}}-6\hat{\mathbf{k}}$

D.

$3\hat{i}-2\hat{j}+\hat{k}$

Answer: C

Solution:

Given that

$$\begin{aligned}
&\mathrm{OP}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}, \mathrm{OQ}=-\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}} \\
&\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}, \mathbf{b}=\hat{\mathbf{j}}-\hat{\mathbf{k}} \\
&l_1=\mathbf{r}_1=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}+\lambda(\hat{\mathbf{i}}+\hat{\mathbf{j}}) \quad \ldots(i) \\
&l_2=\mathbf{r}_2=-\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}+\mu(\hat{\mathbf{j}}-\hat{\mathbf{k}}) \quad \ldots(ii)
\end{aligned}$$

Let point of intersection of first line.

$$\begin{aligned}
&\mathbf{L}=\mathbf{P}+\lambda\mathbf{a}=(1+\lambda)\hat{\mathbf{i}}+(-1+\lambda)\hat{\mathbf{j}}-\hat{\mathbf{k}} \\
&M=\mathbf{Q}+\mu\mathbf{b}=-\hat{\mathbf{i}}+(1+\mu)\hat{\mathbf{j}}+(1-\mu)\hat{k}
\end{aligned}$$

Line passing through L and M

$\therefore \mathbf{LM} \| \mathbf{C}$

$\mathbf{LM}=(-2-\lambda)\hat{\mathbf{i}}+(2+\mu-\lambda)\hat{\mathbf{j}}+(2-\mu)\hat{k}$

$$\begin{aligned}
&\mathbf{LM}=t(\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}) \\
&-2-\lambda=t \quad \ldots(iii) \\
&\Rightarrow \quad \lambda=-2-t \\
&2+\mu-\lambda=-t \quad \ldots(iv) \\
&2-\mu=t \quad \ldots(v) \\
&\Rightarrow \quad \mu=2-t \\
&t=-6, \lambda=4 \\
&\mu=8 \\
&\mathbf{M}=-\hat{\mathbf{i}}+9\hat{\mathbf{j}}-7\hat{\mathbf{k}} \\
&\mathbf{PM}=-2\hat{\mathbf{i}}+10\hat{\mathbf{j}}-6\hat{\mathbf{k}}
\end{aligned}$$

Question28

For $a \in R$, if the vectors $\mathbf{p}=(a+1)\hat{\mathbf{i}}+a\hat{\mathbf{j}}+a\hat{\mathbf{k}}$, $\mathbf{q}=a\hat{\mathbf{i}}+(a+1)\hat{\mathbf{j}}+a\hat{\mathbf{k}}$ and $\mathbf{r}=a\hat{\mathbf{i}}+a\hat{\mathbf{j}}+(a+1)\hat{\mathbf{k}}$ are coplanar and $3(\mathbf{p}\cdot\mathbf{q})^2-\lambda|\mathbf{r}\times\mathbf{q}|^2=0$, then the value of λ is

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Options:

A.

$\frac{2}{3}$

B.

$\frac{3}{2}$

C.

2

D.

1

Answer: D

Solution:

$\mathbf{p} = (a+1)\hat{\mathbf{i}} + a\hat{\mathbf{j}} + a\hat{\mathbf{k}}$
$\mathbf{q} = a\hat{\mathbf{i}} + (a+1)\hat{\mathbf{j}} + a\hat{\mathbf{k}}$
$\mathbf{r} = a\hat{\mathbf{i}} + a\hat{\mathbf{j}} + (a+1)\hat{\mathbf{k}}$

$\mathbf{p}, \mathbf{q}$ and $\mathbf{r}$ are coplanar

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$
$$\Rightarrow (3a+1) \begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$
$$\Rightarrow (3a+1)\left[(a+1)^2 - a^2 - 1\left(a^2 + a - a^2\right) + 1 \left(a^2 - a^2 - a\right)\right] = 0$$

$$\Rightarrow (3a+1)[2a+1-a-a] \Rightarrow a = -\frac{1}{3}$$
$$\mathbf{p} \cdot \mathbf{q} = -\frac{2}{9} - \frac{2}{9} + \frac{1}{9} = \frac{-3}{9} = \frac{-1}{3}$$
$$\Rightarrow |\mathbf{r} \times \mathbf{q}| = \sqrt{\left(\frac{4}{9} - \frac{1}{9}\right)^2 - \left(\frac{-2}{9} - \frac{1}{9}\right)^2 + \left(\frac{1}{9} + \frac{2}{9}\right)^2}$$

$$= \sqrt{\frac{9-9+9}{81}} = \frac{1}{3}$$
$$3(\mathbf{p} \cdot \mathbf{q})^2 - \lambda|\mathbf{r} \times \mathbf{q}|^2 = 0 \Rightarrow \lambda = 1$$

Question29

If $\mathbf{a} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, $\mathbf{b} = -2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\mathbf{c} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ are three vectors such that $(\mathbf{b} \times \mathbf{c}) \times \mathbf{a} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, then $x + y - z =$

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Options:

A.

75

B.

-89

C.

125

D.

-389

Answer: A

Solution:

$$\begin{aligned}
\mathbf{a} &= \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}} \\
\mathbf{b} &= -2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}} \\
\mathbf{c} &= 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}} \\
&\Rightarrow (\mathbf{b} \times \mathbf{c}) \times \mathbf{a} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \\
&\Rightarrow (\mathbf{b} \cdot \mathbf{a})\mathbf{c} - (\mathbf{c} \cdot \mathbf{a})\mathbf{b} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \\
&\Rightarrow (-2 + 20 + 8)\mathbf{c} - (3 - 8 + 16)\mathbf{b} \\
&\quad = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \\
&\Rightarrow 100\hat{\mathbf{i}} + 107\hat{\mathbf{j}} - 82\hat{\mathbf{k}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \\
&\Rightarrow x + y - z = 100 - 107 + 82 = 75
\end{aligned}$$

Question30

If $A = (0, 4, -3), B = (5, 0, 12)$ and $C = (7, 24, 0)$, then $\sqrt{BAC} =$

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Options:

A.

60°

B.

$\cos^{-1}\left(\frac{16}{\sqrt{13}}\right)$

C.

$\cos^{-1}\left(\frac{13}{38}\right)$

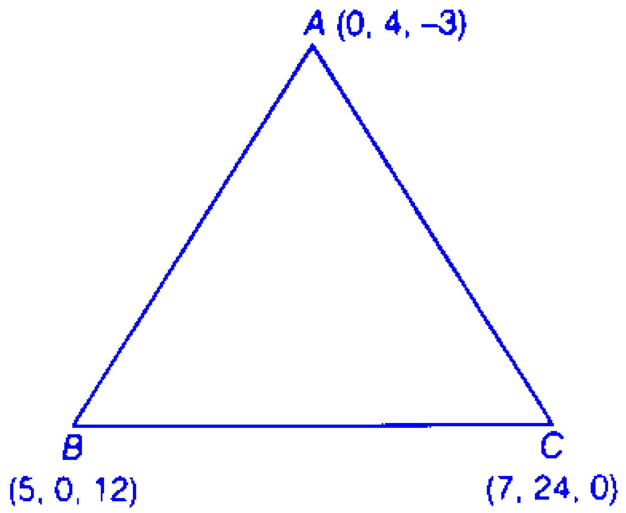
D.

90°

Answer: D

Solution:

$$\begin{aligned}
\mathbf{AB} &= 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 15\hat{\mathbf{k}} \\
\mathbf{AC} &= 7\hat{\mathbf{i}} + 20\hat{\mathbf{j}} + 3\hat{\mathbf{k}}
\end{aligned}$$



$$\cos\theta = \frac{35 - 80 + 45}{|\mathbf{AB}||\mathbf{AC}|} = 0$$
$$\theta = 90^\circ$$

Question31

Let the position vectors of the vertices of a $\triangle ABC$ be $\mathbf{a}, \mathbf{b}, \mathbf{c}$. If on the plane of the triangle, P is a point having position vector $\mathbf{x}$ such that $\mathbf{x} \cdot (\mathbf{c} - \mathbf{b}) = \mathbf{a} \cdot \mathbf{c} - \mathbf{a} \cdot \mathbf{b}$ and $\mathbf{x} \cdot (\mathbf{a} - \mathbf{c}) = \mathbf{ab} - \mathbf{bc}$, then for the $\triangle ABC$, P is the

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Options:

A.

Centroid

B.

Circumcentre

C.

Incentre

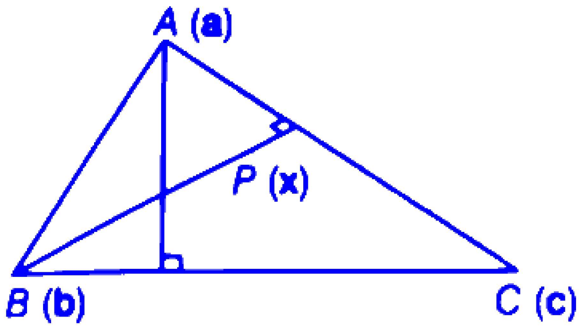
D.

Orthocentre

Answer: D

Solution:

We have, $\mathbf{x} \cdot (\mathbf{c} - \mathbf{b}) = \mathbf{a} \cdot \mathbf{c} - \mathbf{a} \cdot \mathbf{b}$



$= \mathbf{a}(\mathbf{c}-\mathbf{b})$

$(\mathbf{x}-\mathbf{a})\cdot(\mathbf{c}-\mathbf{b})=0$

$\mathbf{AP}\cdot\mathbf{BC}=0$

$\mathbf{AP}\perp\mathbf{BC}$

and $\mathbf{x}\cdot(\mathbf{a}-\mathbf{c})=\mathbf{a}\cdot\mathbf{b}-\mathbf{b}\cdot\mathbf{c}=(\mathbf{a}-\mathbf{c})\mathbf{b}$

$\Rightarrow (\mathbf{x}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{c})=0$

$\mathbf{BP}\cdot\mathbf{AC}=0$

$\therefore \quad \mathbf{BP}\perp\mathbf{AC}$

$\because$ Point of intersection of altitude is called orthocentre.

$\because$ P is orthocentre of $\triangle ABC$.

Question32

$\mathbf{a},\mathbf{b},\mathbf{c}$ are three vectors such that $|\mathbf{a}|=2, |\mathbf{b}|=3, |\mathbf{c}|=5, |\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{69}$. If $(\mathbf{a}\cdot\mathbf{b})=(\mathbf{b}\cdot\mathbf{c})=\frac{\pi}{3}$, then $(\mathbf{c},\mathbf{a})=$

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Options:

A.

$\frac{\pi}{6}$

B.

$\frac{\pi}{4}$

C.

$\frac{\pi}{3}$

D.

$\frac{\pi}{2}$

Answer: C

Solution:

Given $|\mathbf{a}|=2, |\mathbf{b}|=3, |\mathbf{c}|=5$

$(a \cdot b) = \frac{\pi}{3}$

$a \cdot b = 3$ and $b \cdot c = \frac{15}{2}$

Given, $|\mathbf{a}+\mathbf{b}+\mathbf{c}| = \sqrt{69}$

On squaring both sides, we get

$$
\begin{aligned}
&|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2+2(\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}) = 69 \\
\Rightarrow \quad & 4+9+25+2\left(3+\frac{15}{2}+\mathbf{c}\cdot\mathbf{a}\right) = 69 \\
\Rightarrow 2 & \left(\frac{21}{2}+\mathbf{c}\cdot\mathbf{a}\right) = 31 \\
\Rightarrow \quad & 21+2\cdot\mathbf{c} = 31 \\
\Rightarrow \quad & 2(\mathbf{c}\cdot\mathbf{a}) = 10 \\
\Rightarrow \quad & \mathbf{c}\cdot\mathbf{a} = 5 \\
\Rightarrow \quad & |\mathbf{c}||\mathbf{a}|\cos\theta = 5 \\
\Rightarrow \quad & 5\times 2\cos\theta = 5 \\
\Rightarrow \quad & \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}
\end{aligned}
$$

Question33

If the points A, B, C, D with positions vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}, \hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}, \hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ respectively form a tetrahedron, then the angle between the faces ABC and ABD of the tetrahedron is

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Options:

A.

$\cos^{-1}\left(\frac{-4}{\sqrt{29}}\right)$

B.

$\cos^{-1}\left(\frac{-4}{5}\right)$

C.

$\cos^{-1}\left(\frac{3}{5}\right)$

D.

$\cos^{-1}\left(\frac{\sqrt{29}}{\sqrt{33}}\right)$

Answer: A

Solution:

We have, $\mathbf{OA} = \hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$,

$\mathrm{OB} = \hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$\mathrm{OC} = \hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\text{OD} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Let $\mathbf{n}_1$ is the vector normal to the face ABC

$\because \mathbf{n}_1 = \mathbf{AB} \times \mathbf{AC}$

$$\mathbf{n}_1 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & -2 & 3 \\ 0 & -3 & 2 \end{vmatrix}$$
$$= \hat{\mathbf{i}}(-4+9) + \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(0) = 5\hat{\mathbf{i}}$$

Let $\mathbf{n}_2$ be the vector normal to face ABD

$\because \mathbf{n}_2 = \mathbf{AB} \times \mathbf{AD}$

$$n_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & -2 & 3 \\ 1 & 0 & 2 \end{vmatrix} = \hat{\mathbf{i}}(-4) - \hat{\mathbf{j}}(-3) + \hat{\mathbf{k}}(2)$$

$\mathbf{n}_2 = -4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Now, angle between $\mathbf{n}_1$ and $\mathbf{n}_2$

$$\cos\theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|} = \frac{-20}{5\sqrt{29}}$$
$$\theta = \cos^{-1}\left(\frac{-4}{\sqrt{29}}\right)$$

Question34

a, b, c are unit vectors. If a, b are perpendicular vectors, $(\mathbf{a} - \mathbf{c}) \cdot (\mathbf{b} + \mathbf{c}) = 0$ and $\mathbf{c} = l\mathbf{a} + m\mathbf{b} + n(\mathbf{a} \times \mathbf{b})$; (l, m, n are scalars), then $n^2 =$

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Options:

A.

$I^2 + m^2$

B.

$-21m$

C.

$2l - 2m$

D.

$lm + l + m$

Answer: B

Solution:

Given, $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$

$\mathbf{a} \cdot \mathbf{b} = 0$
$(\mathbf{a} - \mathbf{c}) \cdot (\mathbf{b} + \mathbf{c}) = 0$
$\mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c} - \mathbf{c} \cdot \mathbf{b} - |\mathbf{c}|^2 = 0$
$\mathbf{a} \cdot \mathbf{c} - \mathbf{b} \cdot \mathbf{c} = 1$
Also, $\mathbf{c} = l\mathbf{c} + m\mathbf{b} + n(\mathbf{a} \times \mathbf{b})$
$\mathbf{a} \cdot \mathbf{c} = l(\mathbf{a})^2 + m(\mathbf{a} \cdot \mathbf{b}) + n(\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b})$
$\mathbf{a} \cdot \mathbf{c} = l$

Similarly, $\mathbf{b} \cdot \mathbf{c} = m$

$\therefore l - m = 1 \Rightarrow l^2 + m^2 - 2lm = 1$
Now, $(\mathbf{c} - l\mathbf{a} - m\mathbf{b})^2 = (n(\mathbf{a} \times \mathbf{b}))^2$
$\Rightarrow \mathbf{c}^2 + l^2\mathbf{a}^2 + m^2\mathbf{b} - 2m\mathbf{b} \cdot \mathbf{c} + 2lm\mathbf{a} \cdot \mathbf{b} - 2l\mathbf{a} \cdot \mathbf{c} = n^2|\mathbf{a}|^2|\mathbf{b}|^2$
$\Rightarrow 1 + l^2 + m^2 - 2m^2 - 2l^2 = n^2$
$\Rightarrow 1 - l^2 - m^2 = n^2$
$\Rightarrow n^2 = -2lm \quad (\because -2lm = 1 - l^2 - m^2)$

Question35

If $O(0,0,0), A(1,2,1), B(2,1,3)$ and $C(-1,1,2)$ are the vertices of a tetrahedron, then the acute angle between its face OAB and edge BC is

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Options:

A.

$\cos^{-1}\left(\frac{6\sqrt{2}}{5\sqrt{7}}\right)$

B.

$\sin^{-1}\left(\frac{6\sqrt{2}}{5\sqrt{7}}\right)$

C.

$\tan^{-1}\left(\frac{6\sqrt{2}}{5\sqrt{7}}\right)$

D.

$\frac{\pi}{2}$

Answer: B

Solution:

Let $\mathbf{n}$ is the normal to the face OAB

$\because \quad \mathbf{n} = \mathbf{OA} \times \mathbf{OB}$

$$\mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

$\mathbf{n} = \hat{\mathbf{i}}(5) - \hat{\mathbf{j}}(1) + \hat{\mathbf{k}}(-3)$
$\mathbf{n} = 5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

Direction ratio of edge $\mathbf{BC} = -3, 0, -1$

Now, angle between $\mathbf{n}$ and edge $\mathbf{BC}$ is

$$\sin\theta = \left|\frac{-15 - 0 + 3}{\sqrt{35}\sqrt{10}}\right| = \frac{-12}{\sqrt{350}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\therefore \quad \sin\theta = \frac{6\sqrt{2}}{5\sqrt{7}}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{6\sqrt{2}}{5\sqrt{7}}\right)$$

Question36

If the angles between the sides of the $\triangle ABC$ formed by $A(2,3,5), B(-1,3,2)$ and $C(3,5,-2)$ are α, β and γ, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

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Options:

A.

1

B.

2

C.

$\frac{3}{2}$

D.

$\frac{1}{2}$

Answer: B

Solution:

$\mathbf{AB} = -3\hat{\mathbf{i}} - 3\hat{\mathbf{k}}$
$\mathbf{BC} = 4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$
$\mathbf{CA} = -\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

Given α, β and γ are angles between the sides

$$\because \cos\alpha = \frac{\mathbf{AB}\cdot\mathbf{BC}}{|\mathbf{AB}||\mathbf{BC}|} = 0$$
$$\cos\beta = \frac{\mathbf{BC}\cdot\mathbf{CA}}{|\mathbf{BC}||\mathbf{CA}|} = \frac{-36}{6\sqrt{54}} = -\frac{2}{\sqrt{6}}$$
$$\cos\gamma = \frac{\mathbf{CA}\cdot\mathbf{AB}}{|\mathbf{CA}||\mathbf{AB}|} = \frac{-18}{6\sqrt{54}} = \frac{2}{\sqrt{12}}$$
$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = \sqrt{(1-\cos^2\alpha)} + \left(\sqrt{1-\cos^2\beta}\right) + \left(\sqrt{1-\cos^2\gamma}\right)$$
$$= 1 + \frac{2}{6} + \frac{8}{12} = 1 + 1 = 2$$

Question37

Let $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}, 5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}, -13\hat{\mathbf{i}} - 11\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ be the position vectors of three points. A, B and C, respectively. If $\mathbf{AB} = \lambda\mathbf{BC}$ and $\mathbf{AC} = \mu\mathbf{CB}$, then $\lambda + \mu =$

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Options:

A.

1

B.

-1

C.

2

D.

-2

Answer: B

Solution:

We have, $A(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}), B(5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ and $C(-13\hat{\mathbf{i}} - 11\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$,

$$\mathbf{AB} = \lambda\mathbf{BC}$$
$$3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} = \lambda(-18\hat{\mathbf{i}} - 12\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$
$$\because -18\lambda = 3$$
$$\lambda = -\frac{3}{18}$$

Given, $\mathbf{AC} = \mu\mathbf{CB}$

$$-15\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 5\hat{\mathbf{k}} = \mu(18\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$$
$$\mu = -\frac{15}{18}$$
$$\therefore \lambda + \mu = -\frac{3}{18} - \frac{15}{18} = -\frac{18}{18} = -1$$

Question38

a, b are position vectors of the point A and B respectively, C and D are points on the line AB such that $\mathbf{AB}, \mathbf{AC}$ and $\mathbf{BD}, \mathbf{BA}$ are two pairs of like vectors. If $\mathbf{AC} = 3\mathbf{AB}$ and $\mathbf{BD} = 2\mathbf{BA}$, then CD

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Options:

A.

$3b - 4a$

B.

$4\mathbf{a} - 4\mathbf{b}$

C.

$4a - 3b$

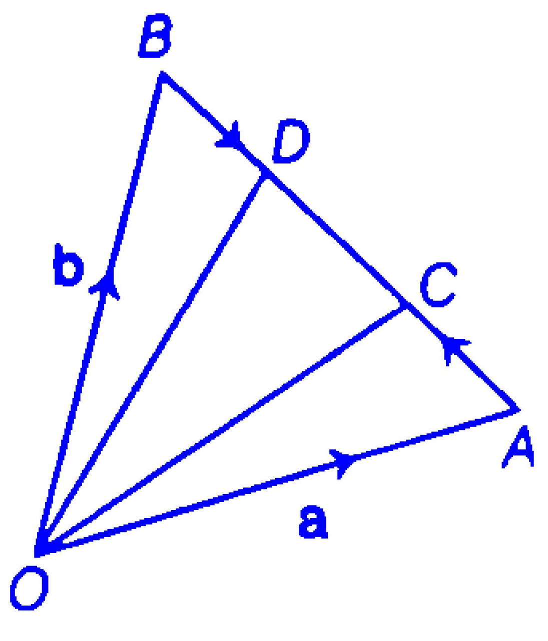
D.

$3b - 3a$

Answer: B

Solution:

$AC = OC - OA$
$OC = OA + AC$
$OC = a + 3AB$
$OC = a + 3(b - a)$
$OC = 3b - 2a$



$$\begin{aligned} \text{Similarly, } \mathbf{BD} &= \mathbf{OD} - \mathbf{OB} \\ \mathbf{OD} &= \mathbf{OB} + \mathbf{BD} \\ &= \mathbf{b} + 2\mathbf{BA} \\ &= \mathbf{b} + 2(\mathbf{a} - \mathbf{b}) \\ \mathbf{OD} &= 2\mathbf{a} - \mathbf{b} \end{aligned}$$

$$\begin{aligned} CD &= OD - OC \\ &= (2a - b) - (3b - 2a) \\ CD &= 4a - 4b \end{aligned}$$

Question39

If a, b, c are three unit vectors such that $|\mathbf{a}-\mathbf{b}|^2+|\mathbf{b}-\mathbf{c}|^2+|\mathbf{c}-\mathbf{a}|^2=15$, then $|\mathbf{a}-\mathbf{b}-\mathbf{c}|^2-4(\mathbf{b}\cdot\mathbf{c})=$

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Options:

A.

6

B.

15

C.

12

D.

10

Answer: C

Solution:

Given,

$$\begin{aligned} & |\mathbf{a}-\mathbf{b}|^2+|\mathbf{b}-\mathbf{c}|^2+|\mathbf{c}-\mathbf{a}|^2=15 \\ \Rightarrow\ & |\mathbf{a}|^2+|\mathbf{b}|^2-2\mathbf{a}\cdot\mathbf{b}+|\mathbf{b}|^2+|\mathbf{c}|^2 \\ & -2\mathbf{b}\cdot\mathbf{c}+|\mathbf{c}|^2+|\mathbf{a}|^2-2\mathbf{c}\cdot\mathbf{a}=15 \\ \Rightarrow\ & 6-2(\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a})=15 \\ \Rightarrow\ & \mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}=-\frac{9}{2} \\ & |\mathbf{a}-\mathbf{b}-\mathbf{c}|^2-4(\mathbf{b}\cdot\mathbf{c}) \\ \Rightarrow\ & |\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2 \\ & +2(-\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}-\mathbf{c}\cdot\mathbf{a})-4(\mathbf{b}\cdot\mathbf{c}) \\ & =3-2(\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}) \\ & =3-2\left(-\frac{9}{2}\right) \\ & =3+9=12 \end{aligned}$$

Question40

If $\mathbf{a} = \hat{\mathbf{i}} + p\hat{\mathbf{j}} - 3\hat{\mathbf{k}}, \mathbf{b} = p\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{c} = -3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are three vectors such that $|\mathbf{a} \times \mathbf{b}| =| \mathbf{a} \times \mathbf{c}$, then $p =$

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Options:

A.

-2

B.

-1

C.

1

D.

2

Answer: D

Solution:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & p & -3 \\ p & -3 & 1 \end{vmatrix}$$

$\hat{\mathbf{i}}(p-9) - \hat{\mathbf{j}}(1+3p) + k\left(-3-p^2\right)$

$$|\mathbf{a} \times \mathbf{b}|^2 = (p-9)^2 + (3p+1)^2 + \left(p^2+3\right)^2$$
$$\mathbf{a} \times \mathbf{c} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & p & -3 \\ -3 & 1 & 2 \end{vmatrix}$$
$$= \hat{\mathbf{i}}(2p+3) - \hat{\mathbf{j}}(2-9) + \hat{\mathbf{k}}(1+3p)$$
$$|\mathbf{a} \times \mathbf{c}|^2 = (2p+3)^2 + 49 + (3p+1)^2$$

Given, $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a} \times \mathbf{c}|$

$$\because \quad (p-9)^2 + (3p+1)^2 + \left(p^2+3\right)^2 = (2p+3)^2 + 49 + (3p+1)^2$$
$$\Rightarrow \quad p^2 + 81 - 18p + \left(p^2+3+2p+3\right)\left(p^2+3-2p-3\right) = 49$$
$$\Rightarrow \quad p^2 + 81 - 18p + \left(p^2+2p+6\right)\left(p^2-2p\right) = 49$$
$$\Rightarrow p^2 + 81 - 18p + \left(p^2+2p+6\right)\left(p^2-2p\right) = 49$$

$\Rightarrow p^4 + 3p^2 - 30p + 32 = 0$
Now put $p = 2.16 + 12 - .60 + 32 = 0$
$\because \quad p = 2$

Question41

If $\mathbf{a} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{c} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{d} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ are four vectors, then $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) =$

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Options:

A.

$17\hat{\mathbf{i}} - 15\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

B.

$31\hat{\mathbf{i}} - \mathbf{j} + 23\hat{\mathbf{k}}$

C.

$17\hat{\mathbf{i}} - \hat{\mathbf{j}} + 23\hat{\mathbf{k}}$

D.

$31\hat{\mathbf{i}} - 15\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

Answer: B

Solution:

$(a \times b) \times (c \times d)$

$$\begin{aligned} &= [\mathbf{abd}]\mathbf{c} - [\mathbf{abc}]\mathbf{d} \\ [\mathbf{abd}]\mathbf{c} &= 8(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \\ &= 24\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 16\hat{\mathbf{k}} \\ [\mathbf{abc}]\mathbf{d} &= -7(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \\ &= -7\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 7\hat{\mathbf{k}} \end{aligned}$$

$[\mathbf{abd}]\mathbf{c} - [\mathbf{abc}]\mathbf{d} = 31\hat{\mathbf{i}} - \hat{\mathbf{j}} + 23\hat{\mathbf{k}}$

Question42

If the vectors $a\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + b\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + c\hat{\mathbf{k}}$ $(a \neq b \neq c \neq 1)$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

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Options:

A. 0

B. 2

C. 1

D. -1

Answer: C

Solution:

Given the vectors $a\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}+b\hat{\mathbf{j}}+\hat{\mathbf{k}}$, and $\hat{\mathbf{i}}+\hat{\mathbf{j}}+c\hat{\mathbf{k}}$, we need to verify that they are coplanar. When vectors are coplanar, it means they can be expressed as a linear combination of the other vectors.

Thus, we can write:

$a\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}=x(\hat{\mathbf{i}}+b\hat{\mathbf{j}}+\hat{\mathbf{k}})+y(\hat{\mathbf{i}}+\hat{\mathbf{j}}+c\hat{\mathbf{k}})$

where x and y are scalars, and not both zero.

Expanding both sides, we get:

$a\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}=(x+y)\hat{\mathbf{i}}+(bx+y)\hat{\mathbf{j}}+(x+cy)\hat{\mathbf{k}}$

Equating the coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, we derive:

$a=x+y$

$1=bx+y$

$1=x+cy$

Now, from Equation (1), we have:

$1-a=1-(x+y)$

Substitute the rearranged form of Equation (2):

$1-b=1-(bx+y)=\frac{x-1+y}{x}$

Substitute from Equation (3):

$1-c=1-(x+cy)=\frac{y-1+x}{y}$

Calculate the desired expression:

$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=\frac{1}{1-(x+y)}+\frac{x}{x-1+y}+\frac{y}{y-1+x}$

By simplifying:

$=\frac{1-x-y}{1-(x+y)}=1$

Thus, the expression simplifies to 1.

Question43

If $\mathbf{AB}=2\mathbf{i}+3\mathbf{j}-6\mathbf{k}, \mathbf{BC}=6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ are the vectors along two sides of a $\triangle ABC$. Then, perimeter of $\triangle ABC$ is

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Options:

A. 21

B. $\sqrt{74}+14$

C. $\sqrt{74}+19$

D. $\sqrt{74}+3$

Answer: B

Solution:

Given, $\mathbf{AB} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

$\mathrm{BC} = 6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

We know that $\mathbf{AB} + \mathbf{BC} + \mathbf{CA} = 0$

$$\begin{aligned} \Rightarrow \quad & (8\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = \mathbf{AC} \\ \therefore \quad & |\mathbf{AB}| = \sqrt{4+9+36} = 7 \\ & |\mathbf{BC}| = \sqrt{36+4+9} = 7 \\ \text{and} \quad & |\mathbf{AC}| = \sqrt{64+1+9} = \sqrt{74} \end{aligned}$$

$\therefore$ Perimeter of $\triangle ABC$ is

$7 + 7 + \sqrt{74} = 14 + \sqrt{74}$

Question44

The orthogonal projection vector of $a = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ on $\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

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Options:

A. $-\frac{1}{6}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

B. $\frac{1}{6}(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

C. $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

D. $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Answer: B

Solution:

To find the orthogonal projection of vector $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ on vector $\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, we use the formula for the projection of $\mathbf{a}$ onto $\mathbf{b}$:

$\text{Projection}_{\mathbf{b}}(\mathbf{a}) = \frac{(\mathbf{a} \cdot \mathbf{b})\mathbf{b}}{|\mathbf{b}|^2}$

Step-by-step, this involves:

Calculate the dot product $\mathbf{a} \cdot \mathbf{b}$:

$\mathbf{a} \cdot \mathbf{b} = (2)(1) + (3)(-2) + (3)(1) = 2 - 6 + 3 = -1$

Find the magnitude squared of b, $|\mathbf{b}|^2$:

$|\mathbf{b}|^2 = (1)^2 + (-2)^2 + (1)^2 = 1 + 4 + 1 = 6$

Substitute these values into the projection formula:

$\text{Projection}_{\mathbf{b}}(\mathbf{a}) = \frac{-1(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})}{6} = \frac{-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}}{6}$

Thus, the orthogonal projection vector of $\mathbf{a}$ on $\mathbf{b}$ is:

$\frac{-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}}{6}$

Question45

If $\mathbf{a}=-4\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $\mathbf{b}=\sqrt{2}\hat{\mathbf{i}}-\sqrt{2}\hat{\mathbf{j}}$ are two vectors, then angle between the vectors $2\mathbf{a}$ and $\frac{\mathbf{b}}{2}$ is

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Options:

A. 30°

B. 135°

C. 90°

D. 0°

Answer: B

Solution:

Given vectors are:

$\mathbf{a}=-4\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

$\mathbf{b}=\sqrt{2}\hat{\mathbf{i}}-\sqrt{2}\hat{\mathbf{j}}$

First, we compute $2\mathbf{a}$ and $\frac{\mathbf{b}}{2}$:

$2\mathbf{a}=-8\hat{\mathbf{i}}+4\hat{\mathbf{j}}+8\hat{\mathbf{k}}$

$\frac{\mathbf{b}}{2}=\frac{1}{\sqrt{2}}\hat{\mathbf{i}}-\frac{1}{\sqrt{2}}\hat{\mathbf{j}}$

To find the angle between $2\mathbf{a}$ and $\frac{\mathbf{b}}{2}$, we use the dot product formula:

$\cos\theta=\frac{2\mathbf{a}\cdot\frac{\mathbf{b}}{2}}{|2\mathbf{a}|\left|\frac{\mathbf{b}}{2}\right|}$

This simplifies to:

$\cos\theta=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

Calculating the dot product $\mathbf{a}\cdot\mathbf{b}$:

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b}&=(-4)(\sqrt{2})+(2)(-\sqrt{2})+(4)(0)\\&=-4\sqrt{2}-2\sqrt{2}\\&=-6\sqrt{2}\end{aligned}$$

Next, we find the magnitudes of $\mathbf{a}$ and $\mathbf{b}$:

$|\mathbf{a}|=\sqrt{(-4)^2+2^2+4^2}=\sqrt{16+4+16}=\sqrt{36}=6$

$|\mathbf{b}|=\sqrt{(\sqrt{2})^2+(-\sqrt{2})^2}=\sqrt{2+2}=\sqrt{4}=2$

Substitute these into the cosine formula:

$\cos\theta=\frac{-6\sqrt{2}}{6\times 2}=\frac{-6\sqrt{2}}{12}=-\frac{1}{\sqrt{2}}$

The angle θ is calculated as:

$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Since $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ corresponds to $\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$:

$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} = 135^\circ$

Question46

A unit vector perpendicular to the vectors $a = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ is

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Options:

A. $\frac{3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{\sqrt{22}}$

B. $\frac{3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}}{\sqrt{22}}$

C. $\frac{3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{\sqrt{22}}$

D. $\frac{3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{\sqrt{22}}$

Answer: B

Solution:

Given vectors $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{b} = 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, we need to find a unit vector that is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. This can be found by determining the cross product $\mathbf{a} \times \mathbf{b}$.

First, let's compute the cross product $\mathbf{a} \times \mathbf{b}$:

$$\begin{aligned}
\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix} \\
&= \hat{\mathbf{i}}(3 \cdot 2 - 4 \cdot 3) - \hat{\mathbf{j}}(2 \cdot 2 - 0 \cdot 4) + \hat{\mathbf{k}}(2 \cdot 3 - 0 \cdot 3) \\
&= \hat{\mathbf{i}}(6 - 12) - \hat{\mathbf{j}}(4 - 0) + \hat{\mathbf{k}}(6 - 0) \\
&= -6\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}
\end{aligned}$$

Next, we need to find the magnitude of the cross product $\mathbf{a} \times \mathbf{b}$:

$|\mathbf{a} \times \mathbf{b}| = \sqrt{(-6)^2 + (-4)^2 + 6^2} = \sqrt{36 + 16 + 36} = \sqrt{88} = 2\sqrt{22}$

Thus, the unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ is derived by normalizing the cross product:

$$\begin{aligned}
\text{Unit vector} &= \pm \frac{-6\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{2\sqrt{22}} \\
&= \frac{-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{\sqrt{22}} \quad \text{or} \quad \frac{3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{\sqrt{22}}
\end{aligned}$$

Therefore, the required unit vectors are $\frac{-3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}}{\sqrt{22}}$ or $\frac{3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}}{\sqrt{22}}$.

Question47

If the vectors $a\hat{\mathbf{i}}+\mathbf{j}+3\hat{\mathbf{k}}, 4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $4\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ are coplanar, then a is equal to

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Options:

A. 2

B. 1

C. 3

D. 4

Answer: A

Solution:

Given that the vectors $a\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}$, $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}$, and $4\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ are coplanar, we can determine the value of a by setting the determinant of their coefficients to zero.

The determinant is:

$$\begin{vmatrix} a & 1 & 3 \\ 4 & 2 & 6 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

Expanding the determinant, we have:

$a(2\times1-6\times5)-1(4\times1-6\times4)+3(4\times5-2\times4)=0$

Calculating each term gives:

$a(2-30)-1(4-24)+3(20-8)=0$

$a(-28)+20+36=0$

This simplifies to:

$-28a+56=0$

Solving for a, we get:

$28a=56 \quad \Rightarrow \quad a=2$

Thus, the value of a is 2.

Question48

Let $|\hat{\mathbf{a}}|=2=|\hat{\mathbf{b}}|=3$ and the angle between $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be $\frac{\pi}{3}$.If a parallelogram is constructed with adjacent sides $2\hat{\mathbf{a}}+3\hat{\mathbf{b}}$ and $\hat{\mathbf{a}}-\hat{\mathbf{b}}$, then its shorter diagonal is of length

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Options:

A. 108

B. 172

C. $6\sqrt{3}$

D. $2\sqrt{43}$

Answer: C

Solution:

Given:

$|\mathbf{a}| = 2$

$|\mathbf{b}| = 3$

Angle $\theta = \frac{\pi}{3}$

The parallelogram has adjacent sides:

$\mathbf{p} = 2\mathbf{a} + 3\mathbf{b}$

$\mathbf{q} = \mathbf{a} - \mathbf{b}$

The diagonals of the parallelogram are described by $\mathbf{p} + \mathbf{q}$ and $\mathbf{p} - \mathbf{q}$.

Calculate $\mathbf{p} + \mathbf{q}$:

$\mathbf{p} + \mathbf{q} = (2\mathbf{a} + 3\mathbf{b}) + (\mathbf{a} - \mathbf{b}) = 3\mathbf{a} + 2\mathbf{b}$

Calculate $|\mathbf{p} + \mathbf{q}|^2$:

$$\begin{aligned} |\mathbf{p} + \mathbf{q}|^2 &= |3\mathbf{a} + 2\mathbf{b}|^2 \\ &= 9|\mathbf{a}|^2 + 4|\mathbf{b}|^2 + 12(\mathbf{a} \cdot \mathbf{b}) \\ &= 9 \times 4 + 4 \times 9 + 12 \times |\mathbf{a}||\mathbf{b}| \cos\theta \\ &= 36 + 36 + 12 \times 2 \times 3 \times \frac{1}{2} \\ &= 36 + 36 + 36 \\ &= 108 \end{aligned}$$

Therefore, the length of the shorter diagonal, $|\mathbf{p} + \mathbf{q}|$, is:

$\sqrt{108} = 6\sqrt{3}$

Question49

The values of x for which the angle between the vectors $x^2\hat{\mathbf{i}} + 2x\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + x\hat{\mathbf{k}}$ is obtuse lie in the interval

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Options:

A. $(-\infty, 0) \cup (3, \infty)$

B. $(0, 3)$

C. $[0, 3]$

D. $(-\infty, 0) \cup 3, \infty)$

Answer: B

Solution:

Given,

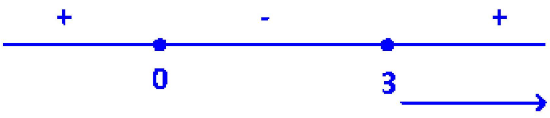
and

$\mathbf{v}_1 = x^2\hat{\mathbf{i}} + 2x\hat{\mathbf{j}} + \hat{\mathbf{k}}$
$\mathbf{v}_2 = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + x\hat{\mathbf{k}}$

The angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is obtuse.

Thus, $\mathbf{v}_1 \cdot \mathbf{v}_2 < 0$

$$\Rightarrow (x^2 - 4x + x) < 0 \Rightarrow x^2 - 3x < 0$$
$$\Rightarrow \quad x(x-3) < 0$$



$x \in (0, 3)$

Question50

Let $\hat{\mathrm{a}} = 3\hat{\mathrm{i}} + 4\hat{\mathrm{j}} - 5\hat{\mathrm{k}}, \hat{\mathrm{b}} = 2\hat{\mathrm{i}} + \hat{\mathrm{j}} - 2\hat{\mathrm{k}}$. The projection d the sum of the vectors $\hat{\mathrm{a}}$ and $\hat{\mathrm{b}}$ on the vector perpendicular to the plance of $\hat{\mathrm{a}}$ and $\hat{\mathrm{b}}$, is

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Options:

A. 0

B. $4\sqrt{2}$

C. $7\sqrt{2}$

D. $\frac{1}{\sqrt{2}}$

Answer: A

Solution:

To find the projection of the sum of vectors $\mathbf{a}$ and $\mathbf{b}$ on a vector perpendicular to their plane, follow these steps:

Calculate $\mathbf{a} + \mathbf{b}$:

$\mathbf{a} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}, \quad \mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\mathbf{a} + \mathbf{b} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$= 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

Find $\mathbf{a} \times \mathbf{b}$ (the cross product):

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 4 & -5 \\ 2 & 1 & -2 \end{vmatrix}$$

$= \hat{\mathbf{i}}(-8+9) - \hat{\mathbf{j}}(-6+10) + \hat{\mathbf{k}}(3-8)$

$= \hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

Normalize the perpendicular vector $\mathbf{n} = -3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$:

First, find the magnitude $|\mathbf{n}|$:

$|\mathbf{n}| = \sqrt{(-3)^2 + (-4)^2 + (-5)^2} = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$

The unit vector $\hat{\mathbf{n}}$ is:

$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{-3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}}{5\sqrt{2}}$

$= \frac{-3}{5\sqrt{2}}\hat{\mathbf{i}} - \frac{4}{5\sqrt{2}}\hat{\mathbf{j}} - \frac{5}{5\sqrt{2}}\hat{\mathbf{k}}$

Projection of $\mathbf{a} + \mathbf{b}$ on $\hat{\mathbf{n}}$:

$(\mathbf{a} + \mathbf{b}) \cdot \hat{\mathbf{n}} = (5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}}) \cdot \left(\frac{-3}{5\sqrt{2}}\hat{\mathbf{i}} - \frac{4}{5\sqrt{2}}\hat{\mathbf{j}} - \frac{5}{5\sqrt{2}}\hat{\mathbf{k}}\right)$

Calculate the dot product, which results in 0. Thus, the projection of the sum of the vectors **a** and **b** on the vector perpendicular to the plane of **a** and **b** is 0.

Question51

In $\triangle PQR$, $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$, $(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ and $(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\mathbf{k})$are the position vectors of the vectices P, Q and R respectively then, the position vector fo the point ol intersection of the angle bisector of P and QR is

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Options:

A. $(6\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 9\hat{\mathbf{k}})$

B. $(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

C. $(5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

D. $\left(\frac{5}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)$

Answer: D

Solution:

In triangle $\triangle PQR$, the vertices have the following position vectors:

$P(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

$Q(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$R(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

The angle bisector of a vertex in a triangle divides the opposite side into segments proportional to the lengths of the other two sides. To find the position vector I of the intersection of the angle bisector from vertex P with side QR, we use the formula:

$I = \frac{|PR| \cdot Q + |PQ| \cdot R}{|PR| + |PQ|}$

First, calculate the lengths of PQ and PR:

$|PQ| = \sqrt{(4-2)^2 + (3-2)^2 + (6-3)^2} = \sqrt{4+1+9} = \sqrt{14}$

$|PR| = \sqrt{(4-3)^2 + (3-1)^2 + (6-3)^2} = \sqrt{1+4+9} = \sqrt{14}$

Since $|PQ| = |PR|$, the intersection point I is the midpoint of Q and R:

$I = \frac{Q+R}{2} = \frac{(2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})+(3\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}})}{2}$

$I = \frac{5\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}}}{2}$

$I = \frac{5}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

Question52

If $\hat{\mathrm{f}} = \hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}$ and $\hat{\mathrm{g}} = 2\hat{\mathrm{i}} - \hat{\mathrm{j}} + 3\hat{\mathrm{k}}$, then the projection vector of $\hat{\mathrm{f}}$ on $\hat{\mathrm{g}}$ is

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Options:

A. $\frac{2}{7}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

B. $\frac{2}{7}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

C. $\frac{1}{3}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

D. $\frac{1}{14}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

Answer: B

Solution:

To find the projection vector of $\hat{\mathbf{f}}$ onto $\hat{\mathbf{g}}$, we start with the given vectors:

$\hat{\mathbf{f}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\hat{\mathbf{g}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

The formula for the orthogonal projection of $\hat{\mathbf{f}}$ on $\hat{\mathbf{g}}$ is:

Projection of $\hat{\mathbf{f}}$ on $\hat{\mathbf{g}} = \frac{\hat{\mathbf{f}} \cdot \hat{\mathbf{g}}}{|\hat{\mathbf{g}}|^2} \cdot \hat{\mathbf{g}}$

First, calculate the dot product $\hat{\mathbf{f}} \cdot \hat{\mathbf{g}}$:

$\hat{\mathbf{f}} \cdot \hat{\mathbf{g}} = (1)(2) + (1)(-1) + (1)(3) = 2 - 1 + 3 = 4$

Next, find the magnitude squared of $\hat{\mathbf{g}}$:

$|\hat{\mathbf{g}}|^2 = (2)^2 + (-1)^2 + (3)^2 = 4 + 1 + 9 = 14$

Now use these results to find the projection:

Projection of $\hat{\mathbf{f}}$ on $\hat{\mathbf{g}} = \frac{4}{14}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

Simplify the fraction:

$= \frac{2}{7}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

Thus, the projection vector of $\hat{\mathbf{f}}$ on $\hat{\mathbf{g}}$ is:

$\frac{2}{7}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

Question53

If θ is the angle between $\hat{\mathbf{f}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\hat{\mathbf{g}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ and $\sin\theta = \sqrt{\frac{24}{28}}$, then $7a^2 + 24a =$

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Options:

A. 10

B. 12

C. 36

D. 15

Answer: A

Solution:

Given the vectors $\mathbf{f} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{g} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + a\hat{\mathbf{k}}$, we know that the sine of the angle θ between them is $\sin\theta = \sqrt{\frac{24}{28}}$.

Since $\sin^2\theta + \cos^2\theta = 1$, we can find $\cos\theta$ as follows:

$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{\frac{1}{7}}$

The cosine of the angle θ is also given by:

$\cos\theta = \frac{\hat{\mathbf{f}} \cdot \hat{\mathbf{g}}}{|\hat{\mathbf{f}}| \cdot |\hat{\mathbf{g}}|}$

Calculating the dot product:

$\hat{\mathbf{f}} \cdot \hat{\mathbf{g}} = (1 \cdot 2) + (2 \cdot -3) + (-3 \cdot a) = 2 - 6 - 3a = -4 - 3a$

Calculating the magnitudes:

$|\hat{\mathbf{f}}| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$

$|\hat{\mathbf{g}}| = \sqrt{2^2 + (-3)^2 + a^2} = \sqrt{13 + a^2}$

Thus, we have:

$\frac{1}{\sqrt{7}} = \frac{-4-3a}{\sqrt{14} \cdot \sqrt{13+a^2}}$

Simplifying, we find:

$2(13 + a^2) = (4 + 3a)^2$

Expanding and rearranging gives:

$26 + 2a^2 = 16 + 9a^2 + 24a$

This simplifies to the quadratic equation:

$7a^2 + 24a - 10 = 0$

Solving for a using the quadratic formula:

$a = \frac{-24 \pm \sqrt{24^2 - 4 \times 7 \times (-10)}}{2 \times 7}$

$a = \frac{-24 \pm \sqrt{576+280}}{14}$

$a = \frac{-24 \pm \sqrt{856}}{14}$

Finally, calculating $7a^2 + 24a$:

Substitute back into the quadratic result:

$7a^2 + 24a = 10$

Question54

If $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}, -3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ are the position vectors of three points, A, B, C respectively, then A, B, C

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Options:

A. are collinear point

B. form an isosceles triangle which is not equilateral

C. form an equilateral triranglé

D. form a scalene triangle

Answer: C

Solution:

To determine the relationship between the points A, B, and C given their position vectors:

$\mathbf{r}_1 = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ for point A

$\mathbf{r}_2 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ for point B

$\mathbf{r}_3 = -3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ for point C

We can calculate the distances between these points:

Distance between A and B:

$$
\begin{aligned}
|\mathbf{AB}| &= |\mathbf{r}_2 - \mathbf{r}_1| \\
&= |(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})| \\
&= |\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}| \\
&= \sqrt{1^2 + 5^2 + 4^2} \\
&= \sqrt{1 + 25 + 16} \\
&= \sqrt{42}
\end{aligned}
$$

Distance between B and C:

$$
\begin{aligned}
|\mathbf{BC}| &= |\mathbf{r}_3 - \mathbf{r}_2| \\
&= |(-3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})| \\
&= | - 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}| \\
&= \sqrt{5^2 + 4^2 + 1^2} \\
&= \sqrt{25 + 16 + 1} \\
&= \sqrt{42}
\end{aligned}
$$

Distance between C and A:

Since

$\mathbf{CA} = \mathbf{r}_1 - \mathbf{r}_3 = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (-3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

you find

$|\mathbf{CA}| = \sqrt{42}.$

All three distances are equal,

$|\mathbf{AB}| = |\mathbf{BC}| = |\mathbf{CA}| = \sqrt{42}$

Thus, points A, B, and C form an equilateral triangle because all three sides are of equal length.

Question55

If a, b, c, d are position vectors of 4 points such that $2a + 3b + 5c - 10d = 0$, then the ratio in which the line joining c and d divides the line segment joining a and b is

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Options:

A. $2 : 3$

B. $-1 : 2$

C. $2 : 1$

D. $3 : 2$

Answer: D

Solution:

Given, the equation,

$$
\begin{aligned}
&2a + 3b + 5c - 10d = 0 \Rightarrow 5e = 10d - 2a - 3b \\
&c = 2d - \frac{2}{5}a - \frac{3b}{5} \qquad \ldots(i)
\end{aligned}
$$

Now, consider the points **a** and **b** with the line joining them divided by point **c** and **d** the position vector of any point dividing **a** and **b** internally in the ratio m : n given by

$P = \frac{m\mathbf{b}+n\mathbf{a}}{m+n}$

Similarly, let the point **c** dividing the line joining **a** and **b** in the ratio $K : 1$.

$\mathbf{c} = \frac{K\mathbf{b}+\mathrm{a}}{K+1} \Rightarrow \frac{K\mathbf{b}+\mathrm{a}}{K+1} = 2\,\mathrm{d} - \frac{2}{5}\mathrm{a} - \frac{3}{5}\,\mathrm{b}$

$\Rightarrow K\mathbf{b} + \mathbf{a} = (K+1)\left(2\mathbf{d} - \frac{2}{5}\mathbf{a} - \frac{3}{5}\mathbf{b}\right)$

$$\Rightarrow K\mathbf{b} + \mathbf{a} = 2(K+1)\mathbf{d} - \frac{2(K+1)}{5}\mathbf{a} - \frac{3(K+1)}{5}\mathbf{b}$$

$$\Rightarrow \left(K + \frac{3(K+1)}{5}\right)\mathbf{b} + \left(1 + \frac{2(K+1)}{5}\right)\mathbf{a} = \frac{3(K+1)}{5}\mathbf{d}$$

$$\Rightarrow \left(\frac{8K+3}{5}\right)\mathbf{b} + \left(\frac{7+2K}{5}\right)\mathbf{a} = 3(K+1)\mathbf{d}$$

On comparing the coefficients and solve for K, we get

$K + 1 = \frac{5}{2}$

Thus, $\frac{K}{1}$ given the ratio in which the point C divides a and b . So, $K : 1 = 3 : 2$

Question56

If a, b, c are 3 vectors such that $|\mathbf{a}| = 5, |\mathbf{b}| = 8, |\mathbf{c}| = 11$ and $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, then the angle between the vectors a and b is

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Options:

A. $\cos^{-1}\frac{2}{5}$

B. $\cos^{-1}\frac{10}{11}$

C. $\cos^{-1}\frac{41}{55}$

D. $\frac{\pi}{3}$

Answer: A

Solution:

Given three vectors **a**, **b**, and **c** with magnitudes $|\mathbf{a}| = 5$, $|\mathbf{b}| = 8$, and $|\mathbf{c}| = 11$, and given that they satisfy the equation $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, we need to determine the angle between vectors **a** and **b**.

First, from the equation $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, we can derive:

$\mathbf{a} + \mathbf{b} = -\mathbf{c}$

By squaring both sides, we have:

$(\mathbf{a} + \mathbf{b})^2 = (-\mathbf{c})^2$

This can be expanded to:

$\mathbf{a}^2 + \mathbf{b}^2 + 2\mathbf{a} \cdot \mathbf{b} = \mathbf{c}^2$

Substituting the magnitudes of the vectors, we get:

$|\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a} \cdot \mathbf{b} = |\mathbf{c}|^2$

$5^2 + 8^2 + 2\mathbf{a} \cdot \mathbf{b} = 11^2$

$25 + 64 + 2\mathbf{a} \cdot \mathbf{b} = 121$

Solving for $2\mathbf{a} \cdot \mathbf{b}$, we find:

$2\mathbf{a} \cdot \mathbf{b} = 121 - 89 = 32$

$\mathbf{a} \cdot \mathbf{b} = \frac{32}{2} = 16$

The cosine of the angle θ between $\mathbf{a}$ and $\mathbf{b}$ is given by:

$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{16}{5\times 8} = \frac{2}{5}$

Thus, the angle between vectors $\mathbf{a}$ and $\mathbf{b}$ is:

$\theta = \cos^{-1}\left(\frac{2}{5}\right)$

Question57

$\mathbf{a} = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \quad \mathbf{b} = \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{c} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ ar linearly dependent vectors and magnitude of $\alpha\sqrt{14}$ If α, β are integers, then $\alpha + \beta =$

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Options:

A. 3

B. -3

C. 5

D. -5

Answer: A

Solution:

$\mathbf{a} = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
$\mathbf{c} = 3\mathbf{i} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, |\mathbf{a}| = \sqrt{14}$

Since, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are linearly dependent the determinant of the matrix formed by these vector must be zero.

$[abc] = 0$

$$\begin{vmatrix} \alpha & \beta & 3 \\ 0 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$\alpha(1-4) - \beta(0-6) + 3(0-3) = 0$

$-3\alpha + 6\beta - 9 = 0$

$-\alpha + 2\beta = 3 \Rightarrow \alpha = 2\beta - 3$

$|\alpha| = \sqrt{14}$

$\alpha^2 + \beta^2 + 9 = 14$

$\alpha^2 + \beta^2 = 14 - 9$

$\alpha^2 + \beta^2 = 5$

$(2\beta - 3)^2 + \beta^2 = 5$

$4\beta^2 + 9 - 12\beta + \beta^2 = 5$

$5\beta^2 - 12\beta + 4 = 0$

$5\beta^2 - (10 + 2)\beta + 4 = 0$

$5\beta^2 - 10\beta - 2\beta + 4 = 0$

$5\beta(\beta - 2) - 2(\beta - 2) = 0$

$(5\beta - 2)(\beta - 2) = 0$

$\beta = 22/5$

On substitute into $-\alpha + 2\beta = 3$

$\beta = 2 - \alpha + 4 = 3$

$\alpha = 1$

$\alpha + \beta = 1 + 2 = 3$

$\beta = \frac{2}{5} \Rightarrow -\alpha + \frac{4}{5} = 3$

On substitute $-\alpha = 3 - \frac{4}{5} = \frac{15-4}{5}$

Neglecting $\alpha = \frac{-11}{5}$

Question58

c is a vector along the bisector of the internal angle between the vectors $\mathbf{a} = 4\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\mathbf{b} = 12\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. If the magnitude of c is $3\sqrt{13}$, then c=

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Options:

A. $5\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 2\sqrt{2\hat{k}}$

B. $10\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$

C. $\mathbf{i} - 10\mathbf{j} + 4\mathbf{k}$

D. $2\sqrt{2}\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

Answer: B

Solution:

Given, $\mathbf{a} = 4\hat{\mathbf{i}} + 7\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and

$\mathbf{b} = 12\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}, |\mathbf{c}| = 3\sqrt{13}$

The required vectors **c** is given by

$$c = \lambda(\mathbf{a}+\mathbf{b})$$
$$= \lambda\left(\frac{\mathbf{a}}{|\mathbf{a}|}+\frac{\mathbf{b}}{|\mathbf{b}|}\right)$$
$$= \lambda\left(\frac{4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-4\hat{\mathbf{k}}}{\sqrt{16+49+16}}+\frac{12\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}}{\sqrt{144+9+16}}\right)$$
$$= \lambda\left[\frac{4\hat{\mathbf{i}}+7\hat{\mathbf{j}}-4\hat{\mathbf{k}}}{9}+\frac{12\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}}{13}\right]$$
$$\mathbf{c} = \lambda\left[\frac{52\hat{\mathbf{i}}+91\hat{\mathbf{j}}-52\hat{\mathbf{k}}+108\hat{\mathbf{i}}-27\hat{\mathbf{j}}+36\hat{\mathbf{k}}}{9\times 13}\right]$$
$$\mathbf{c} = \lambda\left[\frac{160\hat{\mathbf{i}}+64\hat{\mathbf{j}}-16\hat{\mathbf{k}}}{9\times 13}\right]$$
$$\mathbf{c} = \frac{16\lambda}{9\times 13}[10\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}]$$
$$|\mathbf{c}| = \frac{16\lambda}{13\times 9}\sqrt{100+16+1} = \frac{16\lambda}{13\times 9}\sqrt{117}$$
$$\mathbf{c} = 3\sqrt{13} \qquad (given)$$
$$3\sqrt{13} = \frac{16\lambda}{13\times 9}\sqrt{117} \Rightarrow 3\sqrt{13} = \frac{16\lambda}{13\times 9}\sqrt{13\times 9}$$
$$3\sqrt{13} = \frac{16\lambda}{13\times 9}3\sqrt{13}$$
$$\lambda = \frac{13\times 9}{16}$$

On putting in Eq.(i), we get

Hence, $c = 10\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$.

Question59

$\mathbf{a} = \hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}, \mathbf{b} = 2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ are two vectors and c is a unit vectors lying in the plane of a and b. If c is perpendicular to b, then $\mathbf{c}(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}) =$

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Options:

A. 0

B. 5

C. $\frac{1}{\sqrt{21}}$

D. $\frac{2}{\sqrt{21}}$

Answer: B

Solution:

The vector **c** is a unit vector that is also perpendicular to **b**, as expressed by:

$\mathbf{b}\cdot\mathbf{c} = 0$

Additionally, the magnitude of **c** is 1:

$|\mathbf{c}| = 1$

To find $\mathbf{c}$, which lies in the plane formed by the vectors $\mathbf{a}$ and $\mathbf{b}$, we can use the cross product of these vectors:

$$\mathbf{c} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

Calculating the determinant gives us:

$$= \hat{\mathbf{i}}(-1-1) - \hat{\mathbf{j}}(1-2) + \hat{\mathbf{k}}(1+2)$$

This simplifies to:

$$= -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

Finally, we compute the dot product of $\mathbf{c}$ with the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$:

$$c \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = (-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

This yields:

$$= 5$$

Thus, $\mathbf{c}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 5$.

Question60

If $\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}, \mathbf{c} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}. \mathbf{d} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ are four vector, then $(\mathbf{a} \times \mathbf{c}) \times (\mathbf{b} \times \mathbf{d}) =$

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Options:

A. $2\hat{\mathbf{i}} + 19\hat{\mathbf{j}} - 11\hat{\mathbf{k}}$

B. $-8\hat{\mathbf{i}} + 19\hat{\mathbf{j}} - 29\hat{\mathbf{k}}$

C. $2\mathbf{i} + \mathbf{j} - 11\mathbf{k}$

D. $-8\hat{\mathbf{i}} + \hat{\mathbf{j}} - 29\hat{\mathbf{k}}$

Answer: D

Solution:

To solve for $(\mathbf{a} \times \mathbf{c}) \times (\mathbf{b} \times \mathbf{d})$, we will begin by calculating the cross products $\mathbf{a} \times \mathbf{c}$ and $\mathbf{b} \times \mathbf{d}$ separately.

We have the vectors defined as:

$$\mathbf{a} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

$$\mathbf{c} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$$\mathbf{d} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Step 1: Calculate $\mathbf{a} \times \mathbf{c}$

The cross product $\mathbf{a} \times \mathbf{c}$ is determined using the determinant:

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 1 \\ 2 & -3 & -1 \end{vmatrix}$$

Calculating the determinant, we have:

$$\hat{\mathbf{i}}[(-1)(-1)-(1)(-3)]-\hat{\mathbf{j}}[(1)(-1)-(1)(2)]+\hat{\mathbf{k}}[(1)(-3)-(-1)(2)]$$

This simplifies to:

$$\hat{\mathbf{i}}(1+3)-\hat{\mathbf{j}}(-1-2)+\hat{\mathbf{k}}(-3+2)$$

Resulting in:

$$4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

Step 2: Calculate $\mathbf{b}\times\mathbf{d}$

Similarly, compute the cross product $\mathbf{b}\times\mathbf{d}$ using the determinant:

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -2 \\ 2 & 1 & 1 \end{vmatrix}$$

Calculating this determinant, we find:

$$\hat{\mathbf{i}}[(1)(1)-(-2)(1)]-\hat{\mathbf{j}}[(1)(1)-(-2)(2)]+\hat{\mathbf{k}}[(1)(1)-(1)(2)]$$

This gives:

$$\hat{\mathbf{i}}(1+2)-\hat{\mathbf{j}}(1+4)+\hat{\mathbf{k}}(1-2)$$

Resulting in:

$$3\hat{\mathbf{i}}-5\hat{\mathbf{j}}-\hat{\mathbf{k}}$$

Step 3: Calculate $(\mathbf{a}\times\mathbf{c})\times(\mathbf{b}\times\mathbf{d})$

Now use the result from both cross products to find their cross product:

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 3 & -1 \\ 3 & -5 & -1 \end{vmatrix}$$

This determinant is computed as follows:

$$\hat{\mathbf{i}}[(3)(-1)-(3)(-1)]-\hat{\mathbf{j}}[(4)(-1)-(-1)(-1)]+\hat{\mathbf{k}}[(4)(-5)-(3)(3)]$$

Which results in:

$$\hat{\mathbf{i}}(-3+5)-\hat{\mathbf{j}}(-4-1)+\hat{\mathbf{k}}(-20-9)$$

Simplifying, we find:

$$-8\hat{\mathbf{i}}+\hat{\mathbf{j}}-29\hat{\mathbf{k}}$$

Thus, the final result of $(\mathbf{a}\times\mathbf{c})\times(\mathbf{b}\times\mathbf{d})$ is:

$$-8\hat{\mathbf{i}}+\hat{\mathbf{j}}-29\hat{\mathbf{k}}$$

Question61

The angle between the diagonals of the parallelogram whose adjacent sides are $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}, \hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ is

Options:

A. $\cos^{-1}\left(\frac{7}{\sqrt{69}}\right)$

B. $\cos^{-1}\left(\frac{1}{7\sqrt{69}}\right)$

C. $\cos^{-1}\left(\frac{1}{7}\right)$

D. $\cos^{-1}\left(\frac{31}{7\sqrt{69}}\right)$

Answer: D

Solution:

To find the angle between the diagonals of a parallelogram, we start with the given vectors representing the adjacent sides of the parallelogram:

Let:

$\mathbf{a} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

$\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

The diagonal vectors of the parallelogram can be calculated as follows:

First diagonal, c:

$\mathbf{c} = \mathbf{a} + \mathbf{b} = (2+1)\hat{\mathbf{i}} + (4+2)\hat{\mathbf{j}} + (-5+3)\hat{\mathbf{k}} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

Second diagonal, d:

$\mathbf{d} = \mathbf{a} - \mathbf{b} = (2-1)\hat{\mathbf{i}} + (4-2)\hat{\mathbf{j}} + (-5-3)\hat{\mathbf{k}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

Next, we calculate the cosine of the angle θ between the diagonals using the dot product formula:

$\cos\theta = \frac{\mathbf{c}\cdot\mathbf{d}}{|\mathbf{c}||\mathbf{d}|}$

Dot Product:

$\mathbf{c}\cdot\mathbf{d} = (3)(1) + (6)(2) + (-2)(-8) = 3 + 12 + 16 = 31$

Magnitudes of the diagonals:

$|\mathbf{c}| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$

$|\mathbf{d}| = \sqrt{1^2 + 2^2 + (-8)^2} = \sqrt{1 + 4 + 64} = \sqrt{69}$

Substituting these into the formula for $\cos\theta$:

$\cos\theta = \frac{31}{7\sqrt{69}}$

Thus, the angle θ between the diagonals is:

$\theta = \cos^{-1}\left(\frac{31}{7\sqrt{69}}\right)$

Therefore, the angle between the diagonals of the parallelogram is $\cos^{-1}\left(\frac{31}{7\sqrt{69}}\right)$.

Question62

If the points having the position vectors $-i+4j-4k$, $3i+2j-5k$, $-3i+8j-5k$ and $-3i+2j+\lambda k$ are coplanar, then $\lambda=$

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Options:

A. 1

B. 2

C. -2

D. -3

Answer: C

Solution:

To determine if the points with the given position vectors are coplanar, we need to ensure the determinant of the matrix formed by vectors **AB**, **AC**, and **AD** is zero.

Given:

$\mathbf{OA}=-\hat{\mathbf{i}}+4\hat{\mathbf{j}}-4\hat{\mathbf{k}}$

$\mathbf{OB}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

$\mathbf{OC}=-3\hat{\mathbf{i}}+8\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

$\mathbf{OD}=-3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$

Calculate the vectors:

$\mathbf{AB}=\mathbf{OB}-\mathbf{OA}=(3+1)\hat{\mathbf{i}}+(2-4)\hat{\mathbf{j}}+(-5+4)\hat{\mathbf{k}}=4\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$\mathbf{AC}=\mathbf{OC}-\mathbf{OA}=(-3+1)\hat{\mathbf{i}}+(8-4)\hat{\mathbf{j}}+(-5+4)\hat{\mathbf{k}}=-2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$\mathbf{AD}=\mathbf{OD}-\mathbf{OA}=(-3+1)\hat{\mathbf{i}}+(2-4)\hat{\mathbf{j}}+(\lambda+4)\hat{\mathbf{k}}=-2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+(\lambda+4)\hat{\mathbf{k}}$

The determinant for coplanarity is:

$$\begin{vmatrix}4 & -2 & -1\\ -2 & 4 & -1\\ -2 & -2 & \lambda+4\end{vmatrix}=0$$

Expanding the determinant:

$=4((4)(\lambda+4)-(-2)(-1))-(-2)((-2)(\lambda+4)-(-2)(-1))-1((-2)(-2)-(4)(-2))$

$=4(4\lambda+16-2)-2(2\lambda+8-2)-(4+8)$

Simplify:

$=16\lambda+56-4\lambda-20-12=0$

Solving:

$12\lambda+24=0$

$12\lambda=-24$

$\lambda=-2$

Therefore, the value of λ is -2.

Question63

If $|f|=10,|g|=14$ and $|f-g|=15$, then $|f+g|=$

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Options:

A. 367

B. $\sqrt{367}$

C. 400

D. 20

Answer: B

Solution:

Given, $|\mathrm{f}|=10,|\mathrm{~g}|=14$ and

$|f-g|=15$

We know that

$$
\begin{aligned}
& |\mathbf{f}-\mathbf{g}|^2=|\mathbf{f}|^2+|\mathbf{g}|^2-2 Re|\mathbf{f}||\mathbf{g}| \\
& \Rightarrow 225=100+196-2 Re|\mathbf{f}||\mathbf{g}| \\
& \Rightarrow 2 Re|\mathbf{f}||\mathbf{g}|=296-225=71 \\
& \text{Now, } |\mathbf{f}+\mathbf{g}|^2=|\mathbf{f}|^2+|\mathbf{g}|^2+2 Re|\mathbf{f}||\mathbf{g}| \\
& =100+196+71 \\
& =367 \\
& \Rightarrow|\mathbf{f}+\mathbf{g}|=\sqrt{367}
\end{aligned}
$$

Question64

If a, b, c are three vectors such that $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=\sqrt{3}$ and $(a+b-c)^2+(b+c-a)^2+(c+a-b)^2=36$, then $|2a-3b+2c|=$

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Options:

A. 15

B. 25

C. 147

D. 75

Answer: D

Solution:

Given, $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=\sqrt{3}$ and

$(a+b-c)^2+(b+c-a)^2+(c+a-b)^2=36$
$|a|^2+|b|^2+|c|^2+2a\cdot b-2a\cdot c-2b\cdot c$
$+|b|^2+|\mathbf{c}|^2+|\mathbf{a}|^2+2\mathbf{b}\cdot\mathbf{c}-2\mathbf{b}\cdot\mathbf{a}-2\mathbf{c}\cdot\mathbf{a}$
$+|\mathbf{c}|^2+|\mathbf{a}|^2+|\mathbf{b}|^2+2\mathbf{c}\cdot\mathbf{a}-2\mathbf{a}\cdot\mathbf{b}$
$-2b\cdot c=36$
$3\left(|a|^2+|b|^2+|c|^2\right)-2a\cdot b-2a\cdot c$
$-2b\cdot c=36$
$3(3+3+3)-2a\cdot b-2a\cdot c-2b\cdot c=36$
$(\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}+\mathbf{b}\cdot\mathbf{c})=\frac{27-36}{2}=\frac{-9}{2}\quad\ldots\text{(i)}$

Now, $|2\mathbf{a}-3\,\mathrm{b}+2\mathbf{c}|^2$

$=4|a|^2+4|b|^2+4|c|^2-12\mathbf{a}\cdot\mathbf{b}+8\mathbf{a}\cdot\mathbf{c}-12\mathbf{b}\cdot\mathbf{c}$

Also, $|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2\geq 0$

$\Rightarrow|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2+2(\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a})\geq 0$
$\Rightarrow\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}\geq-\frac{9}{2}$

Since, $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}=-\frac{9}{2}$

So, $|\mathbf{a}+\mathbf{b}+\mathbf{c}|=0$
$\Rightarrow\quad a+b+c\quad\ldots\text{(ii)}$
So, $|2a-3b+2c|=|(2a+2c)-3b|$
$|-2b-3b|,d=|-5b|$
$=5|b|=5\sqrt{3}$

$\therefore|2a-3b+2c|^2=(5\sqrt{3})^2=75$

Question65

a, b, c are non-coplanar vectors. If $\alpha\mathbf{d}=\mathbf{a}+\mathbf{b}+\mathbf{c}$ and $\beta\mathbf{a}=\mathbf{b}+\mathbf{c}+\mathbf{d}$, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}|=$

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Options:

A. 1

B. 2

C. $|a-b-c|$

D. 0

Answer: D

Solution:

Given the non-coplanar vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, we start with the equations:

$\mathbf{a}+\mathbf{b}+\mathbf{c}=\alpha\mathbf{d}$

From this equation, we can express $\mathbf{b}+\mathbf{c}$ as:

$\mathbf{b}+\mathbf{c}=\alpha\mathbf{d}-\mathbf{a}\quad$ (Equation i)

We are also given:

$\mathbf{b}+\mathbf{c}+\mathbf{d}=\beta\mathbf{a}$

From which we derive:

$\mathbf{b}+\mathbf{c}=\beta\mathbf{a}-\mathbf{d}$ (Equation ii)

By equating both expressed forms of $\mathbf{b}+\mathbf{c}$ from Equations (i) and (ii), we have:

$\alpha\mathbf{d}-\mathbf{a}=\beta\mathbf{a}-\mathbf{d}$

Rearranging terms gives:

$\alpha\mathbf{d}+\mathbf{d}=\beta\mathbf{a}+\mathbf{a}$

Thus, we have:

$(\alpha+1)\mathbf{d}=(\beta+1)\mathbf{a}$

Since $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are non-coplanar, they are linearly independent. Therefore, the coefficients of $\mathbf{a}$ and $\mathbf{d}$ must separately satisfy the equality:

$1+\beta=0$ implies $\beta=-1$

$1+\alpha=0$ implies $\alpha=-1$

Therefore:

$\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}=0$

Thus, the magnitude is:

$|\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}|=0$

Question66

u, v and w are three unit vectors. Let $\hat{\mathbf{p}}=\hat{\mathbf{u}}+\hat{\mathbf{v}}+\hat{\mathbf{w}}\cdot\hat{\mathbf{q}}=\hat{\mathbf{u}}\times(\hat{\mathbf{v}}\times\hat{\mathbf{w}})$. If $\hat{\mathbf{p}}\cdot\hat{\mathbf{u}}=\frac{3}{2}\cdot\hat{\mathbf{p}}\hat{\mathbf{v}}=\frac{7}{4}|\hat{\mathbf{p}}|=2$ and $v=K.q$, then $K=$

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Options:

A. -1

B. 2

C. 3

D. -2

Answer: B

Solution:

Given that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are unit vectors, we define:

$\hat{\mathbf{p}}=\hat{\mathbf{u}}+\hat{\mathbf{v}}+\hat{\mathbf{w}}$

$\hat{\mathbf{q}}=\hat{\mathbf{u}}\times(\hat{\mathbf{v}}\times\hat{\mathbf{w}})$

Additionally, we know:

$\hat{\mathbf{p}}\cdot\hat{\mathbf{u}}=\frac{3}{2}$

$\hat{\mathbf{p}}\cdot\hat{\mathbf{v}}=\frac{7}{4}$

$|\hat{\mathbf{p}}| = 2$

Calculation Steps:

From the equation $\mathbf{p} \cdot \mathbf{u}$:

$\mathbf{p} \cdot \mathbf{u} = |\mathbf{u}|^2 + \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} = \frac{3}{2}$

This implies:

$1 + \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} = \frac{3}{2}$

$\Rightarrow \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} = \frac{1}{2}$

From the equation $\mathbf{p} \cdot \mathbf{v}$:

$\mathbf{p} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{v} + |\mathbf{v}|^2 + \mathbf{v} \cdot \mathbf{w} = \frac{7}{4}$

Thus:

$\mathbf{u} \cdot \mathbf{v} + 1 + \mathbf{v} \cdot \mathbf{w} = \frac{7}{4}$

$\Rightarrow \mathbf{u} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{w} = \frac{3}{4}$

Equating results:

Solve the system:

$\mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w} = \frac{1}{2}$

$\mathbf{u} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{w} = \frac{3}{4}$

Subtract to find:

$\mathbf{u} \cdot \mathbf{w} - \mathbf{v} \cdot \mathbf{w} = -\frac{1}{4}$

Vector subtraction and dot product:

$\mathbf{w} \cdot (\mathbf{u} - \mathbf{v}) = -\frac{1}{4}$

Given also:

$\mathbf{p} \cdot (\mathbf{u} - \mathbf{v}) = \frac{3}{2} - \frac{7}{4} = -\frac{1}{4}$

Conclude for K:

This means:

$\mathbf{p} = \mathbf{w} \quad \Rightarrow \quad |\mathbf{p}| = |\mathbf{w}| \quad \Rightarrow \quad \mathbf{u} + \mathbf{v} = 0$

For:

$\mathbf{q} = \mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

Expanding:

$\mathbf{q} = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

Knowing $\mathbf{v} = K\mathbf{q}$:

$\Rightarrow \mathbf{u} \cdot \mathbf{v} = 0$

Then solve for K:

$K = \frac{1}{\mathbf{u} \cdot \mathbf{w}}$

From above, $\mathbf{u} \cdot \mathbf{v} = \frac{1}{2}$, hence:

$K = 2$

Question67

If a and b are the two non collinear vectors, then $|b|a + |a|b$ represents

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Options:

A. a vector parallel to an angle bisector of a, b

B. a vector along the difference of the **a**, b

C. **a** vector along $a + b$

D. a vector outside the triangle having a, b as adjacent sides

Answer: A

Solution:

The expression $|\mathbf{b}|\mathbf{a} + |\mathbf{a}|\mathbf{b}$ represents a vector parallel to the angle bisector of the two non-collinear vectors **a** and **b**.

To understand why, consider the angle bisector of the angle between vectors **a** and **b**:

$\lambda(\hat{\mathbf{a}} + \hat{\mathbf{b}})$

This can be rewritten as:

$\lambda\left(\frac{\mathbf{a}}{|\mathbf{a}|} + \frac{\mathbf{b}}{|\mathbf{b}|}\right)$

Simplifying further, we have:

$\frac{\lambda}{|\mathbf{a}||\mathbf{b}|}(|\mathbf{b}|\mathbf{a} + |\mathbf{a}|\mathbf{b}) = k(|\mathbf{b}|\mathbf{a} + |\mathbf{a}|\mathbf{b})$

This derivation shows that $|\mathbf{b}|\mathbf{a} + |\mathbf{a}|\mathbf{b}$ is indeed parallel to the angle bisector of **a** and **b**.

Question68

If LMN are the mid-points of the sides PQ, QR and $RPd \triangle PQR$ respectively, then $\mathbf{QM} + \mathbf{LN} + \mathbf{ML} + \mathbf{RN} - \mathbf{MN} - \mathbf{QL} =$

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Options:

A. $PQ + QR + LM + MN$

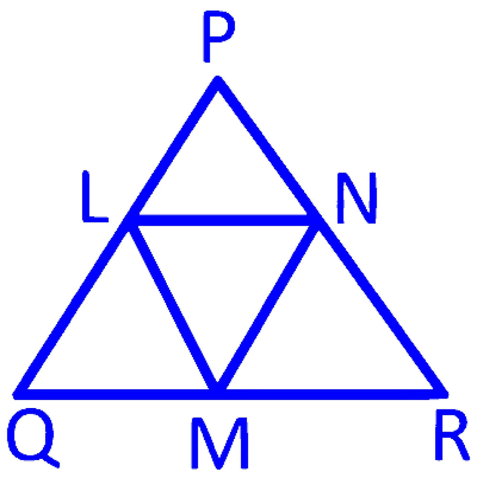
B. $LP + PM + MQ$

C. $PQ + QR - PR$

D. $LM - MN + NR$

Answer: C

Solution:



By mid-point 5 theorem,

$LN = QM = \frac{1}{2} QR$

$$LM = NR = \frac{1}{2} PR$$
$$MN = QL = \frac{1}{2} PQ$$

$$\mathbf{QM} + \mathbf{LN} + \mathbf{ML} + \mathbf{RN} - \mathbf{MN} - \mathbf{QL} =$$
$$= \frac{1}{2}\mathbf{QR} + \frac{1}{2}\mathbf{QR} - \frac{1}{2}\mathbf{PR} - \frac{1}{2}\mathbf{PR} + \frac{1}{2}\mathbf{PQ} + \frac{1}{2}\mathbf{PQ}$$
$$= \mathbf{QR} - \mathbf{PR} + \mathbf{PQ}$$

Question69

Let $\mathbf{a} \times \mathbf{b} = 7\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and $\mathbf{a} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$. If the length of projection of b on a is $\frac{8}{\sqrt{14}}$, then $|b| =$

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Options:

A. 121

B. $\sqrt{11}$

C. $\sqrt{12}$

D. 144

Answer: B

Solution:

The given information states that the projection of vector $\mathbf{b}$ onto vector $\mathbf{a}$ is $\frac{8}{\sqrt{14}}$. Let's go through the calculation step-by-step to find $|\mathbf{b}|$.

First, we express the projection of $\mathbf{b}$ onto $\mathbf{a}$:

$\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \frac{8}{\sqrt{14}}$

From this, we have:

$\frac{\mathbf{a}\cdot\mathbf{b}}{\sqrt{14}} = \frac{8}{\sqrt{14}} \implies \mathbf{a}\cdot\mathbf{b} = 8$

Next, the magnitudes of the vectors satisfy the following relation:

$|\mathbf{a}|^2|\mathbf{b}|^2 = (\mathbf{a}\cdot\mathbf{b})^2 + |\mathbf{a}\times\mathbf{b}|^2$

Given that $\mathbf{a}\times\mathbf{b} = 7\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, the magnitude $|\mathbf{a}\times\mathbf{b}|$ can be calculated as:

$|\mathbf{a}\times\mathbf{b}| = \sqrt{7^2 + (-5)^2 + (-4)^2} = \sqrt{49 + 25 + 16} = \sqrt{90}$

Now, substitute these values into the equation:

$14|\mathbf{b}|^2 = 64 + 90$

$14|\mathbf{b}|^2 = 154$

Therefore:

$|\mathbf{b}|^2 = \frac{154}{14} = 11$

Thus, the magnitude of $\mathbf{b}$ is:

$|\mathbf{b}| = \sqrt{11}$

Question70

Let ABC be an equilateral triangle of side a. M and N are two points on the sides AB and AC, respectively such that $\mathbf{AN} = {}'K\mathbf{AC}$ and $\mathbf{AB} = 3\mathbf{AM}$. If the vectors $\mathbf{BN}$ and $\mathbf{CM}$ are perpendicular, then $K =$

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Options:

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $-\frac{1}{5}$

D. $-\frac{2}{5}$

Answer: A

Solution:

Consider point A as the origin, denoted as O. Let $\mathbf{b}$ and $\mathbf{c}$ be the position vectors of points B and C respectively.

Since the vector $\mathbf{AM} = \frac{\mathbf{AB}}{3} = \frac{\mathbf{b}}{3}$, the position vector of M is $\frac{\mathbf{b}}{3}$.

The position vector of N is given by $\mathbf{AN} = K\mathbf{AC} = K\mathbf{c}$.

Since the vectors $\mathbf{BN}$ and $\mathbf{CM}$ are perpendicular, we use the dot product property for perpendicular vectors:

$(K\mathbf{c} - \mathbf{b}) \cdot \left(\frac{\mathbf{b}}{3} - \mathbf{c}\right) = 0$

Expanding this, we have:

$\frac{K}{3}\mathbf{b} \cdot \mathbf{c} - \frac{|\mathbf{b}|^2}{3} - K|\mathbf{c}|^2 + \mathbf{b} \cdot \mathbf{c} = 0$

Substituting, knowing that for an equilateral triangle, $\mathbf{b} \cdot \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos(60^\circ) = \frac{a^2}{2}$:

$\frac{K}{3} \cdot \frac{a^2}{2} - \frac{a^2}{3} - Ka^2 + \frac{a^2}{2} = 0$

Simplifying the equation:

$|\mathbf{a}|^2 \left[\frac{K}{6} - \frac{1}{3} - K + \frac{1}{2}\right] = 0$

Since $|\mathbf{a}| \neq 0$ (as it represents side length a), the term in square brackets must equal zero:

$\frac{K}{6} - K - \frac{1}{3} + \frac{1}{2} = 0$

This simplifies to:

$\frac{-5K}{6} + \frac{1}{6} = 0$

Thus:

$-5K + 1 = 0 \quad \Rightarrow \quad K = \frac{1}{5}$

Question71

Let $\mathbf{a}$ and $\mathbf{b}$ be two non-collinear vector of unit modulus. If $\mathbf{u} = \mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b}$ and $\mathbf{v} = \mathbf{a} \times \mathbf{b}$, then $|\mathbf{v}| =$

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Options:

A. $|\mathbf{u}| + |\mathbf{u} \cdot \mathbf{v}|$

B. $\frac{|\mathbf{u}|}{2}$

C. $|\mathbf{u}| + \frac{|\mathbf{u} \cdot \mathbf{b}|}{2}$

D. $\frac{|\mathbf{u}|}{5}$

Answer: A

Solution:

To clarify the problem, we start by considering two non-collinear unit vectors, $\mathbf{a}$ and $\mathbf{b}$.

We are given:

$\mathbf{u} = \mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b}$

$\mathbf{v} = \mathbf{a} \times \mathbf{b}$

Let's analyze $|\mathbf{u}|$:

$\mathbf{u} = \mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b}$

The dot product of $\mathbf{u}$ with itself yields:

$|\mathbf{u}|^2 = \mathbf{u} \cdot \mathbf{u} = [\mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b}] \cdot [\mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b}]$

Expanding gives:

$|\mathbf{u}|^2 = \mathbf{a} \cdot \mathbf{a} - 2(\mathbf{a} \cdot \mathbf{b})(\mathbf{b} \cdot \mathbf{a}) + (\mathbf{a} \cdot \mathbf{b})^2(\mathbf{b} \cdot \mathbf{b})$

Given $|\mathbf{a}| = |\mathbf{b}| = 1$, this simplifies to:

$|\mathbf{u}|^2 = 1 - (\mathbf{a} \cdot \mathbf{b})^2$

Since $(\mathbf{a} \cdot \mathbf{b}) = \cos\theta$, we have:

$|\mathbf{u}|^2 = 1 - \cos^2\theta = \sin^2\theta$

Thus, $|\mathbf{u}| = \sin\theta$.

Now, consider $|\mathbf{v}|$:

$\mathbf{v} = \mathbf{a} \times \mathbf{b}$

The magnitude of the cross product is:

$|\mathbf{v}| = |\mathbf{a}||\mathbf{b}|\sin\theta = \sin\theta$

Now let us verify the expression $|\mathbf{u}| + |\mathbf{u} \cdot \mathbf{v}|$:

$$\begin{aligned}|\mathbf{u} \cdot \mathbf{v}| &= |(\mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b}) \cdot (\mathbf{a} \times \mathbf{b})| \\ &= |\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})\mathbf{b} \cdot (\mathbf{a} \times \mathbf{b})|\end{aligned}$$

Using vector triple product identities, we know:

$\mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) = 0$

$\mathbf{b} \cdot (\mathbf{a} \times \mathbf{b}) = 0$

Therefore:

$|\mathbf{u} \cdot \mathbf{v}| = 0$

Thus, $|\mathbf{u}| + |\mathbf{u} \cdot \mathbf{v}| = \sin\theta + 0 = \sin\theta$.

Hence, $|\mathbf{v}| = |\mathbf{u}| + |\mathbf{u} \cdot \mathbf{v}|$.

Question72

In a regular hexagon $ABCDEF$, $\mathbf{AB} = \mathbf{a}$ and $\mathbf{BC} = \mathbf{b}$, then $FA =$

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Options:

A. $\mathbf{a} - \mathbf{b}$

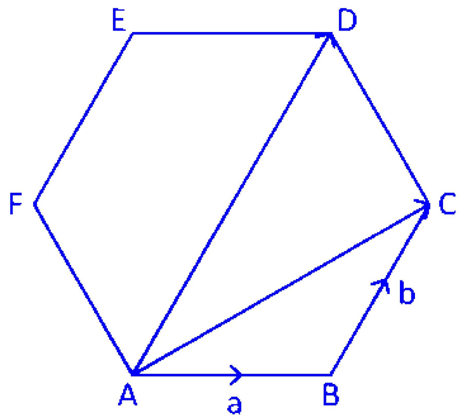
B. $a + b$

C. $\mathbf{b} - \mathbf{a}$

D. $2\mathbf{b} - \mathbf{a}$

Answer: A

Solution:



In $\triangle ABC, AB + BC = AC$

$\Rightarrow \quad AC = a + b$

AD is parallel to BC and also

$AD = 2BC$
$AD = 2b$
$CD = AD - AC = 2b - (a + b) = b - a$

FA is parallel to CD, but is opposite direction.

$\mathrm{FA} = -\mathrm{CD} = -(\mathrm{b} - \mathrm{a}) = \mathrm{a} - \mathrm{b}$

Question73

If $\mathbf{f}, \mathbf{g}, \mathbf{h}$ be mutually orthogonal vectors of equal magnitudes, then the angle between the vectors $\mathbf{f} + \mathbf{g} + \mathbf{h}$ and $\mathbf{h}$ is

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Options:

A. $\cos^{-1}\left(\frac{\sqrt{3}}{4}\right)$

B. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\pi - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D. $\pi - \cos^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Answer: B

Solution:

To find the angle between the vectors $\mathbf{f} + \mathbf{g} + \mathbf{h}$ and $\mathbf{h}$, we start by noting that $\mathbf{f}$, $\mathbf{g}$, and $\mathbf{h}$ are mutually orthogonal vectors of equal magnitude. This means:

$|\mathbf{f}| = |\mathbf{g}| = |\mathbf{h}|.$

$\mathbf{f} \cdot \mathbf{g} = \mathbf{g} \cdot \mathbf{h} = \mathbf{h} \cdot \mathbf{f} = 0.$

The dot product of $(\mathbf{f}+\mathbf{g}+\mathbf{h})$ with $\mathbf{h}$ can be calculated as:

$$(\mathbf{f}+\mathbf{g}+\mathbf{h}) \cdot \mathbf{h}=\mathbf{f} \cdot \mathbf{h}+\mathbf{g} \cdot \mathbf{h}+\mathbf{h} \cdot \mathbf{h}=|\mathbf{h}|^2$$

Let θ be the angle between $\mathbf{f}+\mathbf{g}+\mathbf{h}$ and $\mathbf{h}$. We use the formula for the dot product in terms of angle:

$$|\mathbf{f}+\mathbf{g}+\mathbf{h}||\mathbf{h}| \cos \theta=|\mathbf{h}|^2$$

Since $\mathbf{f}$, $\mathbf{g}$, and $\mathbf{h}$ are orthogonal and of equal magnitude, the magnitude of $\mathbf{f}+\mathbf{g}+\mathbf{h}$ is:

$$|\mathbf{f}+\mathbf{g}+\mathbf{h}|=\sqrt{|\mathbf{f}|^2+|\mathbf{g}|^2+|\mathbf{h}|^2}=\sqrt{3}|\mathbf{h}|$$

Substitute this back into the equation:

$$\sqrt{3}|\mathbf{h}||\mathbf{h}| \cos \theta=|\mathbf{h}|^2$$

Simplifying, we find:

$$\cos \theta=\frac{1}{\sqrt{3}}$$

Therefore, the angle θ is:

$$\theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Question74

Let a, b be two unit vectors. If $\mathbf{c}=\mathbf{a}+2\mathbf{b}$ and $\mathbf{d}=5\mathbf{a}-4\mathbf{b}$ are perpendicular to each other, then the angle between a and b is

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Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{8}$

Answer: C

Solution:

Given two unit vectors $\mathbf{a}$ and $\mathbf{b}$, we know that:

$$|\mathbf{a}|=|\mathbf{b}|=1$$

Consider the vectors $\mathbf{c}=\mathbf{a}+2\mathbf{b}$ and $\mathbf{d}=5\mathbf{a}-4\mathbf{b}$. These vectors are perpendicular, meaning:

$$\mathbf{c} \cdot \mathbf{d}=0$$

Let's compute the dot product:

$$(\mathbf{a}+2\mathbf{b}) \cdot(5\mathbf{a}-4\mathbf{b})=\mathbf{a} \cdot(5\mathbf{a}-4\mathbf{b})+2\mathbf{b} \cdot(5\mathbf{a}-4\mathbf{b})$$

Expanding this, we have:

$$=\mathbf{a} \cdot 5\mathbf{a}-\mathbf{a} \cdot 4\mathbf{b}+2\mathbf{b} \cdot 5\mathbf{a}-2\mathbf{b} \cdot 4\mathbf{b}$$

This simplifies to:

$$=5|\mathbf{a}|^2-4\mathbf{a} \cdot \mathbf{b}+10\mathbf{a} \cdot \mathbf{b}-8|\mathbf{b}|^2$$

Simplifying further by substituting $|\mathbf{a}| = 1$ and $|\mathbf{b}| = 1$:

$= 5(1) + 6\mathbf{a} \cdot \mathbf{b} - 8(1) = 0$

Which simplifies to:

$5 - 8 + 6(\mathbf{a} \cdot \mathbf{b}) = 0$

$-3 + 6(\mathbf{a} \cdot \mathbf{b}) = 0$

Solving for $\mathbf{a} \cdot \mathbf{b}$:

$6(\mathbf{a} \cdot \mathbf{b}) = 3$

$\mathbf{a} \cdot \mathbf{b} = \frac{3}{6} = \frac{1}{2}$

The dot product $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, so:

$\cos \theta = \frac{1}{2}$

Thus, the angle θ between $\mathbf{a}$ and $\mathbf{b}$ is:

$\theta = \frac{\pi}{3}$

Question75

If the vectors $\mathbf{a} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}, \mathbf{c} = 3\hat{\mathbf{i}} + p\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ are coplanar, then $p =$

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Options:

A. 4

B. 14

C. -4

D. 41

Answer: C

Solution:

To determine the value of p for which the vectors $\mathbf{a} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, and $\mathbf{c} = 3\hat{\mathbf{i}} + p\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ are coplanar, we need to evaluate the condition of coplanarity. The vectors are coplanar if the determinant of the matrix formed by their components is zero:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & p & 5 \end{vmatrix} = 0$$

Calculate the determinant:

$= 2\,(2 \cdot 5 - (-3) \cdot p) - (-1)\,(1 \cdot 5 - (-3) \cdot 3) + 1\,(1 \cdot p - 2 \cdot 3)$

Simplify each term:

The first term: $2(10 + 3p) = 20 + 6p$

The second term: $-1(5 + 9) = -14$

The third term: $1(p - 6) = p - 6$

Combining these terms gives:

$20 + 6p - 14 + p - 6 = 0$

Simplifying further:

$20 + 6p - 14 + p - 6 = 0 \implies 7p = -28 \implies p = -4$

Thus, the value of p is -4.

Question76

If (α, β, γ) are the direction cosines of an angular bisector of two lines whose direction ratios are $(2, 2, 1)$ and $(2, -1, -2)$, then $(\alpha + \beta + \gamma)^2 =$

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Options:

A. 3

B. 2

C. 4

D. 5

Answer: B

Solution:

Given the problem involves finding the value of $(\alpha + \beta + \gamma)^2$ when (α, β, γ) are the direction cosines of the angular bisector of two lines with direction ratios $(2, 2, 1)$ and $(2, -1, -2)$.

Finding Direction Cosines:

The direction ratios for line L_1 are $(2, 2, 1)$, and for line L_2 are $(2, -1, -2)$.

The direction cosines for L_1 are found as:

$\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

Similarly, for L_2 the direction cosines are:

$\left(\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$

Perpendicular Check:

The dot product of the two direction ratios confirms that the lines are perpendicular:

$2 \times 2 + 2 \times (-1) + 1 \times (-2) = 0$

Angle Bisector Direction Cosines:

The formulas for the direction cosines of the angle bisectors are:

$\left(\frac{l_1+l_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}}\right)$

and

$\left(\frac{l_1-l_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}\right)$

Calculate the Direction Cosines:

For an angle bisector direction, calculate:

$\left(\frac{4}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}}\right)$

Another possible set is:

$\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Calculation of $(\alpha+\beta+\gamma)^2$:

Using $\alpha = \frac{4}{3\sqrt{2}}, \beta = \frac{1}{3\sqrt{2}}, \gamma = \frac{-1}{3\sqrt{2}}$:

Sum:

$\alpha+\beta+\gamma = \frac{4}{3\sqrt{2}} + \frac{1}{3\sqrt{2}} - \frac{1}{3\sqrt{2}} = \frac{4}{3\sqrt{2}}$

Squaring:

$(\alpha+\beta+\gamma)^2 = \left(\frac{4}{3\sqrt{2}}\right)^2 = \frac{16}{18} = \frac{8}{9}$

Alternatively, use $\alpha = 0, \beta = \frac{1}{\sqrt{2}}, \gamma = \frac{1}{\sqrt{2}}$:

Sum:

$\alpha+\beta+\gamma = 0 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

Squaring:

$(\alpha+\beta+\gamma)^2 = (\sqrt{2})^2 = 2$

Based on calculations with valid assumptions, the value of $(\alpha+\beta+\gamma)^2$ considering a correct scenario for calculation comes out to be 2.

Question77

a, b, c are non-coplanar vectors. If $\mathbf{a}+3\mathbf{b}+4\mathbf{c} = x(\mathbf{a}-2\mathbf{b}+3\mathbf{c}) + y(\mathbf{a}+5\mathbf{b}-2\mathbf{c}) + z(6\mathbf{a}+14\mathbf{b}+4\mathbf{c})$, then $x+y+z=$

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Options:

A. -5

B. -4

C. 4

D. 5

Answer: B

Solution:

$\mathbf{a}+3\mathbf{b}+4\mathbf{c} = \mathbf{a}(x+y+6z) + \mathbf{b}(-2x+5y+14z) + \mathbf{c}(3x-2y+4z)$

(because a, b and c are non-coplanar vectors)

Comparing both sides,

$x+y+6z=1$ .... (i)
$-2x+5y+14z=3$ .... (ii)
$3x-2y+4z=4$ .... (iii)

By solving Eqs. (i), (ii) and (iii), we get

$x=-2, y=-3, z=1$
$x+y+z=-4$

Question78

Three vectors of magnitudes $a, 2a, 3a$ are along the directions of the diagonals of 3 adjacent faces of a cube that meet in a point. Then, the magnitude of the sum of those diagonals is

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Options:

A. 4a

B. 5a

C. 6a

D. 8a

Answer: B

Solution:

Let the vectors of magnitude $a, 2a, 3a$ are along OP, OQ, OR, respectively.

Then, vectors are OP, OQ, OR are $a\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right), 2a\left(\frac{\hat{j}+\hat{k}}{\sqrt{2}}\right), 3a\left(\frac{\hat{k}+\hat{i}}{\sqrt{2}}\right)$, respectively.

Their resultant say R is given by

$$\mathbf{R}=a\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)+2a\left(\frac{\hat{j}+\hat{k}}{\sqrt{2}}\right)+3a\left(\frac{\hat{k}+\hat{i}}{\sqrt{2}}\right)$$
$$=\frac{a}{\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})$$
$$\therefore |\mathbf{R}|=\sqrt{\frac{a^2}{2}(16+9+25)}=5a$$

Question79

If a is collinear with $\mathbf{b}=3\hat{i}+6\hat{j}+6\hat{k}$ and $\mathbf{a}\cdot\mathbf{b}=27$, then $|\mathbf{a}|=$

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Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Let $\mathbf{a}=x\hat{i}+y\hat{j}+z\hat{k}, \mathbf{b}=3\hat{i}+6\hat{j}+6\hat{k}$
$\mathbf{a}\cdot\mathbf{b}=27$

According to question, a is a collinear with b, then

$$\mathbf{a}=\lambda\mathbf{b} \quad \text{... (i)}$$
$$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b}=27$$
$$\Rightarrow \quad \lambda\mathbf{b}\cdot\mathbf{b}=27 \quad \text{[from Eq. (i)]}$$
$$\Rightarrow \quad \lambda|\mathbf{b}|^2=27$$

$$\lambda=\frac{27}{\left(\sqrt{(3)^2+(6)^2+(6)^2}\right)^2}=\frac{27}{(9)^2}$$
$$\lambda=\frac{1}{3}$$
$$\mathrm{a}=\frac{1}{3}(3\hat{i}+6\hat{j}+6\hat{k}) \Rightarrow \mathbf{a}=\hat{i}+2\hat{j}+2\hat{k}$$

$$\text{So, } |\mathbf{a}|=\sqrt{1+(2)^2+(2)^2}$$
$$=\sqrt{9}=3 \text{ units}$$

Question80

Let a, b and c be unit vectors such that a is perpendicular to the plane containing b and c and angle between b and c is $\frac{\pi}{3}$. Then, $|\mathrm{a}+\mathrm{b}+\mathrm{c}|=$

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Options:

A. 3

B. 1

C. 2

D. 4

Answer: C

Solution:

As given $\mathbf{a}$ is perpendicular to $\mathbf{b}$ and $\mathbf{c}$

$\Rightarrow \mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{a} \cdot \mathbf{c} = 0$ and angle between $\mathbf{b}$ and $\mathbf{c} = \frac{\pi}{3}$

$\therefore \mathbf{b} \cdot \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos \frac{\pi}{3} = 1 \cdot 1 \cdot \frac{1}{2}$

$= \frac{1}{2}$ [$\because$ b and c are unit vectors]

Now, $|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = |a|^2 + |b|^2 + |c|^2 + 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a})$

$$= 1 + 1 + 1 + 2\left(0 + \frac{1}{2} + 0\right) = 1 + 1 + 1 + 1 = 4$$

$|a + b + c| = \sqrt{4} = 2$ units

Question81

Let $\mathbf{F} = 2\hat{i} + 2\hat{j} + 5\hat{k}, A = (1, 2, 5), B = (-1, -2, -3)$ and $\mathbf{BA} \times \mathbf{F} = 4\hat{i} + 6\hat{j} + 2\lambda\hat{k}$, then $\lambda =$

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Options:

A. 0

B. 1

C. 2

D. -2

Answer: D

Solution:

Given,

$$\mathbf{F} = 2i + 2j + 5k, A = (1, 2, 5)$$
$$B = (-1, -2, -3)$$
$$\mathbf{BA} = \mathbf{A} - \mathbf{B} = (\hat{i} + 2\hat{j} + 5\hat{k}) - (-\hat{i} - 2\hat{j} - 3\hat{k})$$
$$\mathbf{BA} = 2\hat{i} + 4\hat{j} + 8\hat{k}$$
$$\mathbf{BA} \times \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 8 \\ 2 & 2 & 5 \end{vmatrix}$$
$$= \hat{i}(20 - 16) - \hat{j}(10 - 16) + \hat{k}(4 - 8)$$
$$= 4\hat{i} + 6\hat{j} - 4\hat{k}$$

So, $4\hat{i} + 6\hat{j} - 4\hat{k} = 4\hat{i} + 6\hat{j} + 2\lambda\hat{k}$

$$\because \quad 2\lambda = -4$$
$$\lambda = -2$$

Question82

$OABC$ is a tetrahedron. If D, E are the mid-points of OA and BC respectively, then $\mathbf{DE} =$

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Options:

A. $\frac{1}{2}(OA + OB + OC)$

B. $\frac{1}{2}(OA + OB - OC)$

C. $\frac{1}{2}(OA - OB + OC)$

D. $\frac{1}{2}(-OA + OB + OC)$

Answer: D

Solution:

Given:

Tetrahedron $OABC$

- D is midpoint of OA

$$\Rightarrow \vec{OD} = \frac{1}{2}\vec{OA}$$

- E is midpoint of BC

$$\Rightarrow \vec{OE} = \frac{1}{2}(\vec{OB} + \vec{OC})$$

Find $\vec{DE}$

$$\vec{DE} = \vec{OE} - \vec{OD}$$

Substitute:

$$\vec{DE} = \frac{1}{2}(\vec{OB} + \vec{OC}) - \frac{1}{2}\vec{OA}$$

Simplify:

$$\vec{DE} = \frac{1}{2}(-\vec{OA} + \vec{OB} + \vec{OC})$$

✅ Final Answer:

$$\boxed{\vec{DE} = \frac{1}{2}(-\vec{OA} + \vec{OB} + \vec{OC})}$$

Question83

If $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ and $|\mathbf{a}| = 7, |\mathbf{b}| = 5, |\mathbf{c}| = 3$ then the angle between $\mathbf{b}$ and $\mathbf{c}$ is

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Options:

A. 30°

B. 45°

C. 60°

D. 90°

Answer: C

Solution:

$$\begin{aligned}
&\text{Given, } \mathbf{a}+\mathbf{b}+\mathbf{c}=0 \text{ and} \\
&|\mathbf{a}|=7,|\mathbf{b}|=5,|\mathbf{c}|=3 \\
\Rightarrow & \mathbf{b}+\mathbf{c}=-\mathbf{a} \Rightarrow|\mathbf{b}+\mathbf{c}|^2=|-\mathbf{a}|^2 \\
\Rightarrow & |\mathbf{b}|^2+|\mathbf{c}|^2+2 \cdot \mathbf{b} \cdot \mathbf{c}=|\mathbf{a}|^2 \\
\Rightarrow & 25+9+2 \cdot \mathbf{b} \cdot \mathbf{c}=49 \\
\Rightarrow & 2 \mathbf{b}=15 \Rightarrow 2|\mathbf{b}| \cdot|\mathbf{c}| \cos \theta=15 \\
\Rightarrow & \cos \theta=\frac{1}{2}=\cos 60^{\circ} \\
\therefore & \theta=60^{\circ}
\end{aligned}$$

Question84

If P and Q are two points on the curve $y=2^{x+2}$ in the rectangular cartesian coordinate system such that $\mathrm{OP} \cdot \hat{i}=-1, \mathrm{OQ} \cdot \hat{i}=2$, then $\mathrm{OQ}-4 \mathrm{OP}=$

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Options:

A. $3 \hat{i}+8 \hat{j}$

B. $4 \hat{i}+6 \hat{j}$

C. $6 \hat{i}+8 \hat{j}$

D. $4 \hat{i}+3 \hat{j}$

Answer: C

Solution:

Given, $\mathrm{OP} \cdot \hat{i}=-1 \Rightarrow P_x=-1$

P lies on $y=2^{x+2}$

$\therefore \quad y=2^{-1+2}=2$

Thus, $(Px, Py)=(-1,2)$

and also given

$OQ \cdot \hat{i}=2 \Rightarrow Q_x=2$

Qlies on $y=2^{x+2}$

$$\therefore \quad y=16$$
$$\therefore \quad (Q_x, Q_y) = (2,16)$$

Thus $\mathbf{OP}=-\hat{i}+2\hat{j}$ and $\mathbf{OQ}=2\hat{i}+16\hat{j}$

OQ $-4\mathbf{OP}=2\hat{i}+16\hat{j}+4\hat{i}-8\hat{j}=6\hat{i}+8\hat{j}$

Question85

In quadrilateral $ABCD$, $\mathbf{AB}=\mathbf{a}, \mathbf{BC}=\mathbf{b}. \mathbf{DA}=\mathbf{a}-\mathbf{b}$, M is the mid-point of BC and X is a point on DM such that, $\mathbf{DX}=\frac{4}{5}$ DM.Then, the points A, X and C.

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Options:

A. form an equilateral triangle.

B. are collinear

C. form an isosceles triangle

D. form a right angled triangle

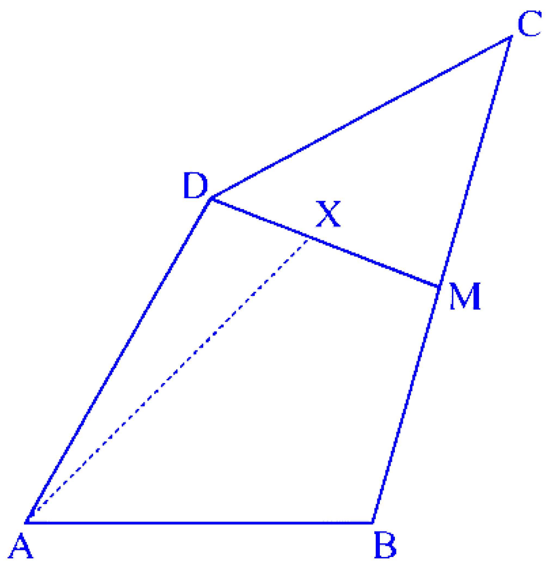
Answer: B

Solution:

$\mathrm{AB}=\mathrm{a}, \mathrm{BC}=\mathbf{b}, \mathrm{DA}=\mathbf{a}-\mathbf{b}$

In $\triangle ABC$, using vector concept, we have

$$\mathbf{AB}+\mathbf{BC}=\mathbf{AC}$$
$$\Rightarrow \quad \mathbf{AC}=\mathbf{a}+\mathbf{b} \quad \text{.... (i)}$$



In ,\triangle A D C$$,

$$\begin{aligned} & AD + DC = AC \\ \Rightarrow \quad & DC = AC - AD \\ & = a + b - (b - a) = 2a \end{aligned}$$

In $\triangle MDC$,

$$\begin{aligned} & MD + DC = MC \\ \Rightarrow & MD = \frac{b}{2} - 2a \end{aligned}$$

[$\because M$ is mid-point of BC]

In $\triangle ADX$,

$$\begin{aligned} & \mathbf{AD} + \mathbf{DX} = \mathbf{AX} \\ & \mathbf{AD} + \frac{4}{5}\mathbf{DM} = \mathbf{AX} \\ \Rightarrow \quad & \mathbf{AX} = \mathbf{b} - \mathbf{a} + \frac{4}{5}\left(2\mathbf{a} - \frac{\mathbf{b}}{2}\right) \\ & = \mathbf{b} - \mathbf{a} + \frac{8}{5}\mathbf{a} - \frac{2}{5}\mathbf{b} = \frac{3}{5}(\mathbf{a} + \mathbf{b}) \\ \Rightarrow \quad & \mathbf{AX} = \frac{3}{5}\mathbf{AC} \quad \text{[from Eq. (i)]} \end{aligned}$$

which says that $\mathbf{AX} \| \mathbf{AC}$ and one point A is common there.

Therefore, A, X and C are collinear.

Question86

The vectors $3\mathbf{a} - 5\mathbf{b}$ and $2\mathbf{a} + \mathbf{b}$ are mutually perpendicular and the vectors $a + 4b$ and $-\mathbf{a} + \mathbf{b}$ are also mutually perpendicular, then the acute angle between a and b is

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Options:

A. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{9}{5\sqrt{43}}\right)$

C. $\pi-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. $\pi-\cos^{-1}\left(\frac{9}{5\sqrt{43}}\right)$

Answer: A

Solution:

$$(3a-5b)\perp(2a+b)$$
$$\Rightarrow (3a-5b)(2a+b)=0$$
$$\Rightarrow 6a\cdot a+3a\cdot b-10b\cdot a-5b\cdot b=0$$
$$\Rightarrow 6|a|^2+3a\cdot b-10a\cdot b-5|b|^2=0$$
$$\Rightarrow 6|a|^2-5|b|^2=7a\cdot b \quad \text{.... (i)}$$

Also, $(a+4b)\perp(-a+b)$

$$\Rightarrow (a+4b)\cdot(-\mathbf{a}+\mathbf{b})=0$$
$$\Rightarrow -\mathbf{a}\cdot\mathbf{a}+\mathbf{a}\cdot\mathbf{b}-4\mathbf{b}\cdot\mathbf{a}+4\mathbf{b}\cdot\mathbf{b}=0$$
$$\Rightarrow -|\mathbf{a}|^2+\mathbf{a}\cdot\mathbf{b}-4\mathbf{a}\cdot\mathbf{b}+4|\mathbf{b}|^2=0$$
$$\Rightarrow -|\mathbf{a}|^2+4|\mathbf{b}|^2=3\mathbf{a}\cdot\mathbf{b} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{1}{7}(6|\mathbf{a}|^2-5|\mathbf{b}|^2)=\frac{1}{3}(-|\mathbf{a}|^2+4|\mathbf{b}|^2)$$
$$\Rightarrow 18|\mathbf{a}|^2-15|\mathbf{b}|^2=-7|\mathbf{a}|^2+28|\mathbf{b}|^2$$
$$\Rightarrow 25|\mathbf{a}|^2=43|\mathbf{b}|^2$$
$$\because \quad 3\mathbf{a}\cdot\mathbf{b}=-|\mathbf{a}|^2+4|\mathbf{b}|^2 \quad \text{[from Eq. (ii)]}$$
$$3|\mathbf{a}||\mathbf{b}|\cos\theta=-\frac{43}{25}|\mathbf{b}|^2+4|\mathbf{b}|^2$$
$$\Rightarrow 3|\mathbf{a}||\mathbf{b}|\cos\theta=\frac{57}{25}|\mathbf{b}|^2$$
$$\Rightarrow 3\sqrt{\frac{43}{25}}||\mathbf{b}|^2\cos\theta=\frac{57}{25}|\mathbf{b}|^2$$
$$\Rightarrow \frac{\sqrt{43}}{5}\cos\theta=\frac{19}{25}\Rightarrow\cos\theta=\frac{19}{5\sqrt{43}}$$
$$\Rightarrow \theta=\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$$

Question87

Let $\mathbf{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $x=2y$. If $|\mathbf{a}|=5\sqrt{2}$ and a makes an angle of 135° with the Z-axis, then $\mathbf{a}=$

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Options:

A. $2\sqrt{3}\hat{i}+\sqrt{3}\hat{j}-3\hat{k}$

B. $2\sqrt{6}\hat{i}+\sqrt{6}\hat{j}-6\hat{k}$

C. $2\sqrt{5}\hat{i}+\sqrt{5}\hat{j}-5\hat{k}$

D. $2\sqrt{5}\hat{i}+\sqrt{5}\hat{j}+5\hat{k}$

Answer: C

Solution:

$\mathbf{a}=x\hat{i}+y\hat{j}+z\hat{k},$

$x=2y$ and $|\mathbf{a}|=5\sqrt{2}$

$\therefore \quad x=|a|\cos\alpha$

$\Rightarrow \quad x=5\sqrt{2}\cos\alpha,$

$y=5\sqrt{2}\cos\beta$

and $z=5\sqrt{2}\cos\gamma=5\sqrt{2}\cos(135^{\circ})$

$=5\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)=-5$

Now, $x=2y$

Now, $x=2y$

$\Rightarrow \quad 5\sqrt{2}\cos\alpha=2\times5\sqrt{2}\cos\beta$

$\Rightarrow \quad \cos\alpha=2\cos\beta$

As, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \quad 4\cos^2\beta+\cos^2\beta+\frac{1}{2}=1$

$\Rightarrow \quad \cos^2\beta=\frac{1}{10}$

$\Rightarrow \quad \cos\beta=\frac{1}{\sqrt{10}}$

$\therefore \quad x=2y=2(5\sqrt{2}\cos\beta)=2\left(5\sqrt{2}\times\frac{1}{\sqrt{10}}\right)=2\sqrt{5}$

$\therefore \quad y=\frac{x}{2}=\sqrt{5}$

$\therefore \quad a=2\sqrt{5}\hat{i}+\sqrt{5}\hat{j}-5\hat{k}$

Question88

Let a, b, c be the position vectors of the vertices of a $\triangle ABC$. Through the vertices, lines are drawn parallel to the sides to form the $\Delta A'B'C'$. Then, the centroid of $\Delta A'B'C'$ is

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Options:

A. $\frac{a+b+c}{9}$

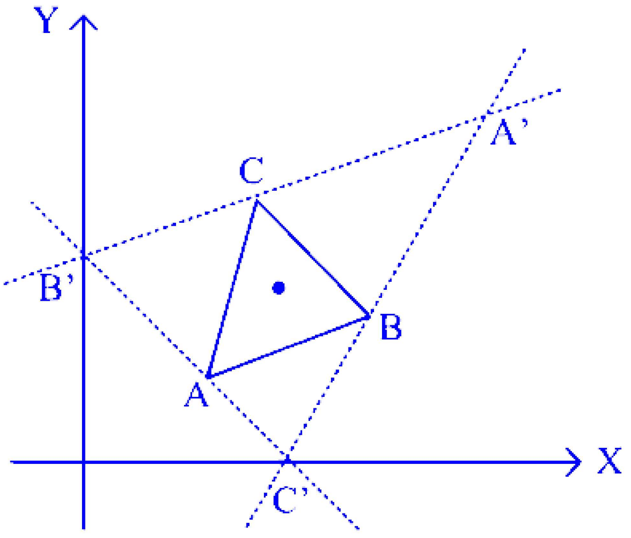
B. $\frac{a+b+c}{6}$

C. $\frac{a+b+c}{3}$

D. $\frac{2(a+b+c)}{3}$

Answer: C

Solution:



Position vectors of the vertices of the $\triangle ABC$ are $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$.

$\therefore$ Centroid of $\triangle ABC$ is $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$

We know, when through the vertices, lines are drawn parallel to the sides to form a new triangle, then obtained such triangle will have the same centroid as the original triangle, which is also clear from the above graphical figure.

$\therefore$ Centroid of $\triangle A'B'C'$ is $\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$.

Question89

If $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{c} = x\hat{\mathbf{i}} + (x-2)\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and if the vector c lies in the plane of vectors a and b and then x equals

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Options:

A. 0

B. 1

C. 2

D. -2

Answer: D

Solution:

$$\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$
$$\mathbf{c} = x\hat{\mathbf{i}} + (x-2)\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

Given, $\mathbf{c}$ lies in the plane of $\mathbf{a}$ and $\mathbf{b}$.

Then, $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = 0$

$$\Rightarrow \quad [\mathbf{a}, \mathbf{b}, \mathbf{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

$$1(1-2(x-2)) - 1(-1-2x) + 1(x-2+x) = 0$$
$$\Rightarrow \quad 1 - 2x + 4 + 1 + 2x + x - 2 + x = 0$$
$$\Rightarrow \quad 2x + 4 = 0$$
$$\Rightarrow \quad x = -2$$

Question90

Let $u = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $v = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}}$. Consider three points P, Q and R having the position vectors $\left(\frac{5}{2}\right)\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$; $\left(\frac{7}{3}\right)\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\left(\frac{9}{4}\right)\hat{\mathbf{i}}$ respectively.Among these, the points in the line passing through u and v are

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Options:

A. Only P and Q

B. Only P and R

C. Only Q and R

D. All P, Q and R

Answer: A

Solution:

The equation of line passing through u and v is $= \vec{u} + \lambda(\vec{v} - \vec{u})$

$$\mathbf{r} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \lambda(\hat{\mathbf{i}} - 6\hat{\mathbf{j}})$$
$$= (2+\lambda)\hat{\mathbf{i}} + (1-6\lambda)\hat{\mathbf{j}}$$

For the point P,

$$\frac{5}{2}\hat{\mathbf{i}} - 2\hat{\mathbf{j}} = (2+\lambda)\hat{\mathbf{i}} + (1-6\lambda)\hat{\mathbf{j}}$$
$$\Rightarrow \quad 2 + \lambda = \frac{5}{2} \Rightarrow \lambda = \frac{1}{2}$$
$$\text{and} \quad 1 - 6\lambda = 1 - 6 \times \frac{1}{2} = -2$$

Thus, point P lie in the line through u and v.

For the point Q

$$2 + \lambda = 7/3 \Rightarrow \lambda = 1/3$$
$$\text{and} \quad 1 - 6\lambda = -1$$

Thus, point Q also lie in the line passes through u and v.

Question91

The point of intersection of the lines joining points $\hat{\mathbf{i}}+2\hat{\mathbf{j}}, 2\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $-\hat{\mathbf{i}}, 2\hat{\mathbf{i}}$ is

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Options:

A. $\frac{5}{3}\hat{\mathbf{i}}$

B. $\frac{3\hat{\mathbf{i}}+\hat{\mathbf{j}}}{5}$

C. $\frac{-3}{5}\hat{\mathbf{i}}$

D. $\frac{2}{5}\hat{\mathbf{j}}$

Answer: A

Solution:

Points $(\hat{\mathbf{i}}+2\hat{\mathbf{j}}),(2\hat{\mathbf{i}}-\hat{\mathbf{j}})$ i.e. $(1,2)$ and $(2,-1)$.

Line joining points $(1,2)$ and $(2,-1)$ is

$y-2=\frac{-1-2}{2-1}(x-1)$

or $\quad y-2=-3x+3$

$y+3x=5$ ..... (i)

Points $(-\hat{\mathbf{i}}),(2\hat{\mathbf{i}})$ i.e. $(-1,0),(2,0)$

Line joining points $(-1,0)(2,0)$ is

$y-0=\frac{0-0}{2+1}(x+1)$

$\Rightarrow \quad y=0$ ..... (ii)

Solve Eqs. (i) and (ii) for x and y, we get

$y=0, x=5/3$

$\therefore$ Point of intersection $\left(\frac{5}{3},0\right)$,

i.e. $\frac{5}{3}\hat{\mathbf{i}}+0\hat{\mathbf{j}}=\frac{5}{3}\hat{\mathbf{i}}$

Question92

The value of $\frac{(\mathbf{a}\times\mathbf{b})^2+(\mathbf{a}\cdot\mathbf{b})^2}{2(\mathbf{a})^2(\mathbf{b})^2}$ is

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Options:

A. 0

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: C

Solution:

$$\frac{(\mathbf{a}\times\mathbf{b})^2+(\mathbf{a}\cdot\mathbf{b})^2}{2|\mathbf{a}|^2|\mathbf{b}|^2}$$
$$=\frac{(|\mathbf{a}|\cdot|\mathbf{b}|\sin\theta\hat{\mathbf{n}})^2+(|\mathbf{a}||\mathbf{b}|\cos\theta)^2}{2|\mathbf{a}|^2|\mathbf{b}|^2}$$
$$=\frac{|\mathbf{a}|^2|\mathbf{b}|^2\sin^2\theta\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}+|\mathbf{a}|^2|\mathbf{b}|^2\cos^2\theta}{2|\mathbf{a}|^2|\mathbf{b}|^2}$$
$$=\frac{\sin^2\theta+\cos^2\theta}{2}=\frac{1}{2}$$

Question93

Let $\mathbf{a}=\hat{\mathbf{i}}-\hat{\mathbf{j}}, \mathbf{b}=\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\mathbf{c}=\hat{\mathbf{k}}-\hat{\mathbf{i}}$ if $\mathbf{d}$ is a unit vector such $\mathbf{a}\cdot\mathbf{b}=0=[\mathbf{bcd}]$, then $\mathbf{d}$ is

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Options:

A. $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

B. $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$

C. $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

D. $\pm\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

Answer: B

Solution:

$\mathbf{a}=\hat{\mathbf{i}}-\hat{\mathbf{j}}, \mathbf{b}=\hat{\mathbf{j}}-\hat{\mathbf{k}}, \mathbf{c}=\hat{\mathbf{k}}-\hat{\mathbf{i}}$

Let $\mathbf{d}=p\hat{\mathbf{i}}+q\hat{\mathbf{j}}+r\hat{\mathbf{k}}$

Given, $\mathbf{a}\cdot\mathbf{d}=0$

$\Rightarrow(\hat{\mathbf{i}}-\hat{\mathbf{j}})\cdot(p\hat{\mathbf{i}}+q\hat{\mathbf{j}}+r\hat{\mathbf{k}})=0$
$\Rightarrow p-q=0\Rightarrow p=q$

Given, $[\mathbf{bcd}]=0$

$$\Rightarrow \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ p & p & r \end{vmatrix}=0$$
$$\Rightarrow (-1)(-r-p)-1(-p)=0$$
$$\Rightarrow r+p+p=0$$
$$\Rightarrow r=-2p$$
$$\therefore \mathbf{d}=p\hat{\mathbf{i}}+p\hat{\mathbf{j}}-2p\hat{\mathbf{k}}$$

Also, $|\mathbf{d}|=1$

$$\Rightarrow \quad \sqrt{p^2+p^2+(-2p)^2}=1$$
$$\Rightarrow \quad 6p^2=1$$
$$\Rightarrow \quad p=\pm\frac{1}{\sqrt{6}}$$
$$\therefore \quad d=\frac{\pm\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{\sqrt{6}}$$

Question94

Let u and v be two non-zero vectors in R^3 with the intermediate angle 45°. Then $|\mathbf{u}\times\mathbf{v}|$ is equal to

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Options:

A. $|u||v|$

B. $2|u||v|$

C. $u\cdot v$

D. $|u|+|v|$

Answer: C

Solution:

$$\begin{aligned} |\mathbf{u}\times\mathbf{v}| &= ||\mathbf{u}|\cdot|\mathbf{v}|\sin\theta\mathbf{n}| \\ &= ||\mathbf{u}|\cdot||\mathbf{v}|\sin 45^\circ\hat{\mathbf{n}}\,| \\ &= ||\mathbf{u}|\cdot|\mathbf{v}\left|\cdot\frac{1}{\sqrt{2}}\hat{\mathbf{n}}\right| \\ &= |\mathbf{u}|\cdot|\mathbf{v}|\cos 45^\circ\cdot 1=\mathbf{u}.\mathbf{v} \end{aligned}$$

Question95

Given, $\mathbf{a} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}, \mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = \mathbf{b}_1 + \mathbf{b}_2$ where $\mathbf{b}_1$ is parallel to a and $\mathbf{b}_2$ is perpendicular to a. Then, $\mathbf{b}_2$ is equal to

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Options:

A. $\frac{1}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

B. $\frac{1}{2}\hat{\mathbf{i}} - \frac{3}{2}\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

C. $\frac{1}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

D. $\frac{1}{2}\hat{\mathbf{i}} - \frac{3}{2}\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

Answer: A

Solution:

$\mathbf{a} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}, \mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

Let $\mathbf{b}_1 = k(3\hat{\mathbf{i}} - \hat{\mathbf{j}})\Big], \mathbf{b}_2 = p\hat{\mathbf{i}} + q\hat{\mathbf{j}} + r\hat{\mathbf{k}}$

$\mathbf{b} = \mathbf{b}_1 + \mathbf{b}_2$

$2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}} = k(3\hat{\mathbf{i}} - \hat{\mathbf{j}}) + (p\hat{\mathbf{i}} + q\hat{\mathbf{j}} + r\hat{\mathbf{k}})$

$2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}} = (3k + p)\hat{\mathbf{i}} + (-k + q)\hat{\mathbf{j}} + r\hat{\mathbf{k}}$

$\Rightarrow \quad 3k + p = 2 - k + q = 1$

and $r = -3 \quad .....(i)$

Given that, $\mathbf{a} \cdot \mathbf{b}_2 = 0$

$\Rightarrow \quad 3p - q = 0 \quad .....(ii)$

Using Eq. (ii) in Eq. (i) $3p = q$, we obtain

$3k + p = 2$
$3p - k = 1$

Solve for k and p, we get

$$k = \frac{1}{2}, p = \frac{1}{2}$$

$$\because \quad q = 3p \Rightarrow q = \frac{3}{2}$$

$$\mathbf{b}_2 = \frac{1}{2}\hat{\mathbf{i}} + \frac{3}{2}\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

Question96

The position vectors of the points A and B with respect to O are $2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. The length of the internal bisector of $\angle BOA$ of $\triangle AOB$ is (take proportionality constant is 2)

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Options:

A. $\frac{\sqrt{136}}{9}$

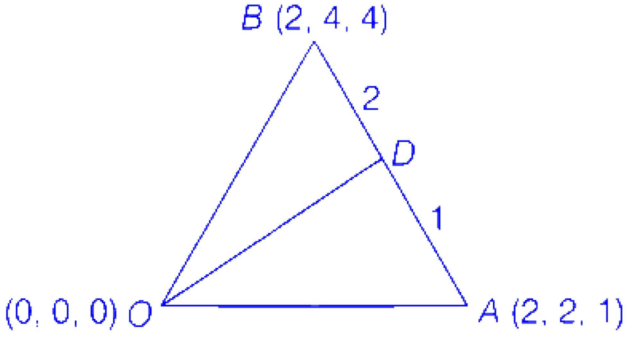
B. $\frac{\sqrt{136}}{3}$

C. $\frac{20}{3}$

D. $\frac{25}{3}$

Answer: B

Solution:



$$AO = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$OB = \sqrt{2^2 + 4^2 + 4^2} = 6$$

$$\frac{AO}{BO} = \frac{AD}{BD} = \frac{3}{6} \quad \text{[Angle bisector theorem]}$$

$$D = \frac{2(2,2,1) + 1(2,4,4)}{2+1} = \frac{(6,8,6)}{3}$$

$$= \left(2, \frac{8}{3}, 2\right)$$

$$(OD) = \sqrt{2^2 + \left(\frac{8}{3}\right)^2 + 2^2} = \frac{\sqrt{136}}{3}$$

Question97

Let $\mathbf{u} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{v} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\mathbf{w} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. Then which of the following statement is true?

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Options:

A. u is perpendicular to v but not w

B. v is perpendicular to w but not u

C. w is perpendicular to u but not v

D. u is perpendicular to both v and w

Answer: A

Solution:

Given,

$$\begin{aligned} \mathbf{u} &=< 2,3,1 > \\ \mathbf{v} &=< -3,2,0 > \\ \mathbf{w} &=< 1,-1,4 > \\ \mathbf{u}\cdot\mathbf{v} &= -6+6+0=0 \\ \mathbf{u}\cdot\mathbf{w} &= 2-3+4=3\neq 0 \end{aligned}$$

$\mathbf{u}\perp\mathbf{v}$ but $\mathbf{u}$ is not perpendicular to $\mathbf{w}$.

Question98

If a = (1, 1, 0) and b = (1, 1, 1), then unit vector in the plane of a and b and perpendicular to a is

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Options:

A. (0, 1, 0)

B. (1, −1, 0)

C. k

D. (1, 0, 1)

Answer: C

Solution:

$$\begin{aligned} &\mathbf{a}=(1,1,0), \\ &\mathbf{b}=(1,1,1) \Rightarrow 0=(\mathbf{a}+\lambda\mathbf{b})\cdot\mathbf{a} \\ &\Rightarrow \quad \mathbf{a}\cdot\mathbf{a}+\lambda\mathbf{a}\cdot\mathbf{b}=0 \\ &\Rightarrow \quad (1+1+0)+\lambda(1+1+0)=0 \\ &\Rightarrow \quad \lambda=-1 \\ &\mathbf{c}=\mathbf{a}+\lambda\mathbf{b}=(0,0,1)=\mathbf{k} \end{aligned}$$

Question99

Let $\mathbf{a} = \hat{\mathbf{i}}$ and $\mathbf{b} = \hat{\mathbf{j}}$, the point of intersection of the lines $\mathbf{r} \times \mathbf{a} = \mathbf{b} \times \mathbf{a}$ and $\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$ is

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Options:

A. $\mathbf{r} = \hat{i} + \hat{j}$

B. $\mathbf{r} = \hat{i} - \hat{j}$

C. $\mathbf{r} = \hat{k}$

D. $\mathbf{r} = 2\hat{i} + \hat{j}$

Answer: A

Solution:

$\mathbf{a} = \hat{\mathbf{i}}, \mathbf{b} = \hat{\mathbf{j}}$
$(\mathbf{r} \times \mathbf{a}) = (\mathbf{b} \times \mathbf{a}) \Rightarrow (\mathbf{r} \times \mathbf{a}) - (\mathbf{b} \times \mathbf{a}) = 0$
$(\mathbf{r} - \mathbf{b}) \times \mathbf{a} = 0 \Rightarrow \mathbf{r} - \mathbf{b} = \lambda \mathbf{a}$
$\mathbf{r} = \mathbf{b} + \lambda \mathbf{a}$
$= \lambda \hat{\mathbf{i}} + \hat{\mathbf{j}} \quad \ldots\ldots\ldots\text{(i)}$
$\mathbf{r} \times \mathbf{b} = \mathbf{a} \times \mathbf{b}$
$\Rightarrow \quad (\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$
$\mathbf{r} = \mathbf{a} + \alpha \mathbf{b}$
$= \hat{\mathbf{i}} + \propto \hat{\mathbf{j}} \quad \ldots\ldots\ldots\text{(ii)}$

Comparing Eqs. (i) and (ii), we get

$\lambda \hat{\mathbf{i}} + \hat{\mathbf{j}} = \hat{\mathbf{i}} + \alpha \hat{\mathbf{j}} \Rightarrow \lambda = 1, \alpha = 1$
$\mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

Question100

Which of the following vector is equally inclined with the coordinate axes?

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Options:

A. $\hat{i} + 2\hat{j} + 3\hat{k}$

B. $2\hat{i} - 2\hat{j} + \hat{k}$

C. $3\hat{i} + 3\hat{j} - 3\hat{k}$

D. $4\hat{i} + 4\hat{j} + 4\hat{k}$

Answer: D

Solution:

In option (d) it is clearly that dr' s of $4\hat{\mathbf{i}}+4\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ is 4, 4, 4

Which makes equal angles with coordinate axes.

Question101

If $\hat{\mathbf{i}}+4\hat{\mathbf{j}}+3\hat{\mathbf{k}}, \hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$, and $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ are position vectors of A, B and C respectively and if D and E are mid points of sides BC and AC, then $\mathbf{DE}$ is equal to

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Options:

A. $\hat{i}+\hat{j}+\hat{k}$

B. $\hat{i}+\hat{j}$

C. $\hat{j}$

D. $\hat{j}+\hat{k}$

Answer: C

Solution:

Position vector of $A=\mathbf{OA}=\hat{\mathbf{i}}+4\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

Position vector of $B=\mathbf{OB}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

Position vector of $C=\mathbf{OC}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Position vector of $D=\mathbf{OD}=$ mid point of $\mathbf{OB}$ and $\mathbf{OC}=\frac{\mathbf{OB}+\mathbf{OC}}{2}$

$=2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

Position vector of $E=\mathbf{OE}=$ mid point of $\mathbf{OA}$

and
$$\begin{aligned} \mathbf{OC} &= \frac{\mathbf{OA}+\mathbf{OC}}{2} \\ &= 2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}} \\ \mathbf{DE} &= \mathbf{OE}-\mathbf{OD}=\hat{\mathbf{j}} \end{aligned}$$

Question102

If a and b are two vectors such that $\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}<0$ and $|\mathbf{a}\cdot\mathbf{b}|=|\mathbf{a}\times\mathbf{b}|$ then the angle between the vectors a and b is

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Options:

A. $\frac{\pi}{4}$

B. $\sec^{-1}(-\sqrt{2})$

C. $\tan^{-1}\left(\frac{-1}{2}\right)$

D. $\sin^{-1}\left(\frac{1}{2}\right)$

Answer: B

Solution:

$$\because \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\cdot|\mathbf{b}|} < 0$$
$$\Rightarrow \mathbf{a}\cdot\mathbf{b} < 0$$

Let θ be the angle between $\overline{\mathbf{a}}$ and $\overline{\mathbf{b}}$, then

$$\begin{aligned} & |\mathbf{a}\cdot\mathbf{b}| = |\mathbf{a}\times\mathbf{b}| \\ \Rightarrow \quad & -|\mathbf{a}|\cdot|\mathbf{b}|\cos\theta = |\mathbf{a}|\mathbf{b}\mid\sin\theta \\ \Rightarrow \quad & \tan\theta = -1 \\ \Rightarrow \quad & \sec\theta = -\sqrt{2} \\ \Rightarrow \quad & \theta = \sec^{-1}(-\sqrt{2}) \end{aligned}$$

Question103

Let a, b and c be three-unit vectors and $\mathbf{a}\cdot\mathbf{b} = \mathbf{a}\cdot\mathbf{c} = 0$. If the angle between b and c is $\frac{\pi}{3}$. Then $[\mathbf{abc}]^2$ is equal to

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Options:

A. $\frac{3}{2}$

B. $\frac{3}{4}$

C. $\frac{2}{3}$

D. $\frac{4}{3}$

Answer: B

Solution:

$\mathbf{a}, \mathbf{b}, \mathbf{c}$ are unit vectors

$\Rightarrow \quad |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$

$\because \quad \mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{a} \cdot \mathbf{c} = 0$

$\Rightarrow \quad \mathbf{a} \perp \mathbf{b}$ and $\mathbf{a} \perp \mathbf{c}$

Angle between $\mathbf{b}$ and $\mathbf{c}$ is $\frac{\pi}{3}$

$\therefore \quad \mathbf{b} \times \mathbf{c} = |\mathbf{b}| \cdot |\mathbf{c}| \sin \frac{\pi}{3} \cdot \hat{\mathbf{a}}$

$[\because \mathbf{a}$ is perpendicular to $\mathbf{b}$ and $\mathbf{c}$ and $\hat{\mathbf{a}} = 1]$

$= 1 \cdot 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Now, $[\mathbf{abc}]^2 = [\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})]^2$

$$= \left[\mathbf{a} \cdot \frac{\sqrt{3}}{2}\right]^2 = |\mathbf{a}|^2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= 1 \cdot \frac{3}{4} = \frac{3}{4}$$

Question104

Let x and y are real numbers. If $\mathbf{a} = (\sin x)\hat{\mathbf{i}} + (\sin y)\hat{\mathbf{j}}$ and $\mathbf{b} = (\cos x)\hat{\mathbf{i}} + (\cos y)\hat{\mathbf{j}}$, then $|\mathbf{a} \times \mathbf{b}|$ is

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Options:

A. 0

B. greater than one

C. less than or equal to 1

D. less than 1

Answer: C

Solution:

$$\mathbf{a} = (\sin x)\hat{\mathbf{i}} + (\sin y)\hat{\mathbf{j}}$$
$$\mathbf{b} = (\cos x)\hat{\mathbf{i}} + (\cos y)\hat{\mathbf{j}}$$
$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \sin x & \sin y & 0 \\ \cos x & \cos y & 0 \end{bmatrix}$$
$$= \hat{\mathbf{k}}(\sin x \cos y - \cos x \sin y)$$
$$\mathbf{a} \times \mathbf{b} = \hat{\mathbf{k}} \sin(x - y)$$
$$\text{or } \mid \mathbf{a} \times \mathbf{b} \mid = \sin(x - y) \leq 1 \quad [\because -1 < \sin x \leq 1, \forall x]$$

Question105

A vector makes equal angles α with X and Y-axis, and 90Υ with Z-axis. Then, α is equal to(c) 45Yand 135Y(d) 90Y

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Options:

A. 60Υ or 120Υ

B. 30Υ or 150Υ

C. 45Υ or 135Υ

D. 90Υ

Answer: C

Solution:

$l=\cos \alpha$

$m=\cos \alpha$

$n=\cos 90 \gamma=0$

$\because l^2+m^2+n^2=1$

$\Rightarrow 2 \cos ^2 \alpha=1$

$\Rightarrow \cos ^2 \alpha=\frac{1}{2}$

$\Rightarrow \cos \alpha= \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \alpha=45 \Upsilon$ or 135°

Question106

Angle made by the position vector of the point $(5,-4,-3)$ with the positive direction of X-axis is

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Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: C

Solution:

Let $A(5, -4, -3)$

Direction ratios of $\mathbf{OA} = 5, -4, -3$

Direction ratios of X-axis $= 1, 0, 0$

$$\begin{aligned} \text{Angle, } \cos\theta &= \frac{5 \cdot 1 + (-4) \cdot 0 + (-3) \cdot 0}{\sqrt{5^2 + (-4)^2 + (-3)^2}\sqrt{1^2 + 0^2 + 0^2}} \\ &= \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}} \\ \therefore \quad \theta &= \frac{\pi}{4} \end{aligned}$$

Question107

If the volume of the parallelopiped formed by the vectors $\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{j}} + a\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}} + \hat{\mathbf{k}}$ becomes minimum, then a is equal to

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Options:

A. $\frac{1}{3}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{2}{\sqrt{3}}$

D. $\frac{2}{3}$

Answer: B

Solution:

Let $\mathbf{p} = \mathbf{i} + a\mathbf{j} + \mathbf{k}$

$\mathbf{q} = \hat{\mathbf{j}} + a\hat{\mathbf{k}} \Rightarrow \mathbf{r} = a\hat{\mathbf{i}} + \hat{\mathbf{k}}$

$$= \mathbf{p} \cdot (\mathbf{q} \times \mathbf{r})$$

$$V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \quad V = a^3 - a + 1$$

Differentiating V with respect to a

$\frac{dV}{da} = 3a^2 - 1$

Again, differentiating w.r.t. a

$\frac{d^2V}{da^2} = 6a$

For stationary points

$$\frac{dV}{da} = 0$$
$$\Rightarrow \quad 3a^2 - 1 = 0$$

or

$a^2 = \frac{1}{3}$ or $a = \pm\frac{1}{\sqrt{3}}$

At,

$$a = \frac{1}{\sqrt{3}},$$
$$\frac{d^2V}{da^2} = 6 \cdot \frac{1}{\sqrt{3}} > 0$$

$\therefore$ Volume is minimum at $a = \frac{1}{\sqrt{3}}$

Question108

If $\mathbf{a} = \frac{3}{2}\hat{\mathbf{k}}$ and $\mathbf{b} = \frac{2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}}{2}$, then angle between $\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-\mathbf{b}$ is

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Options:

A. 45ϒ

B. 90ϒ

C. 30ϒ

D. 60ϒ

Answer: B

Solution:

$$\mathbf{a} = \frac{3}{2}\hat{\mathbf{k}}, \mathbf{b} = \frac{2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}}{2}$$
$$\mathbf{a}+\mathbf{b} = \frac{3}{2}\hat{\mathbf{k}} + \frac{2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}}{2} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$
$$\mathbf{a}-\mathbf{b} = \frac{3}{2}\hat{\mathbf{k}} - \frac{2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}}{2} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

Let θ be the angle between $(\mathbf{a}+\mathbf{b})$ and $(\mathbf{a}-\mathbf{b})$ then

$$\cos\theta = \frac{(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})}{|\mathbf{a}+\mathbf{b}||\mathbf{a}-\mathbf{b}|}$$
$$= \frac{1(-1) + 1(-1) + 1(2)}{\sqrt{1^2+1^2+1^2}\sqrt{(-1)^2+(-1)^2+2^2}} = 0$$
$$\therefore \quad \theta = 90ϒ$$

Question109

Let $\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, \mathbf{b}=\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ and $\mathbf{c}=7\hat{\mathbf{i}}+9\hat{\mathbf{j}}+11\hat{\mathbf{k}}$, then the area of parallelogram having diagonals $\mathbf{a}+\mathbf{b}$ and $\mathbf{b}+\mathbf{c}$ is

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Options:

A. $4\sqrt{6}$ sq units

B. $2\sqrt{6}$ sq units

C. $\sqrt{6}$ sq units

D. $6\sqrt{6}$ sq units

Answer: A

Solution:

$$\mathbf{a}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$$
$$\mathbf{b}=\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$$
$$\mathbf{c}=7\hat{\mathbf{i}}+9\hat{\mathbf{j}}+11\hat{\mathbf{k}}$$

Then, $\mathbf{a}+\mathbf{b}=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ and $\mathbf{b}+\mathbf{c}=8\hat{\mathbf{i}}+12\hat{\mathbf{j}}+16\hat{\mathbf{k}}=4(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})$ Area of parallelogram $=\frac{1}{2}|(\mathbf{a}+\mathbf{b})\times(\mathbf{b}+\mathbf{c})|$

$$=\frac{1}{2}\mid 2(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})\times 4(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})$$
$$=\frac{8}{2}|(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})\times(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})|$$
$$=4\left\|\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 2 & 3\\ 2 & 3 & 4\end{vmatrix}\right\|=4|-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}|$$
$$=4\sqrt{1+4+1}=4\sqrt{6}\text{ sq units}$$

Question110

If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $|\mathbf{a}|=2, |\mathbf{b}|=3$ and $\mathbf{a}+t\mathbf{b}$ and $\mathbf{a}-t\mathbf{b}$ are perpendicular, where t is a positive scalar, then

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Options:

A. $t=\pm\frac{2}{3}$

B. $t=\frac{4}{9}$

C. $t=\frac{2}{3}$

D. $t = \frac{2}{9}$

Answer: C

Solution:

$$|\mathbf{a}| = 2, |\mathbf{b}| = 3$$
$$\because \quad (\mathbf{a} + t\mathbf{b}) \perp (\mathbf{a} - t\mathbf{b})$$
$$\Rightarrow \quad (\mathbf{a} + t\mathbf{b}) \cdot (\mathbf{a} - t\mathbf{b}) = 0$$
$$\Rightarrow \quad \mathbf{a} \cdot \mathbf{a} - t(\mathbf{a} \cdot \mathbf{b}) + t(\mathbf{b} \cdot \mathbf{a}) - t^2(\mathbf{b} \cdot \mathbf{b}) = 0$$
$$\Rightarrow \quad |\mathbf{a}|^2 - t^2|b|^2 = 0$$
$$\Rightarrow \quad t^2 = \frac{|\mathbf{a}|^2}{|\mathbf{b}|^2}$$
$$\Rightarrow \quad t = \frac{|\mathbf{a}|}{|\mathbf{b}|} = \frac{2}{3}$$
